AP Physics 1 – Summer Tutorial 1

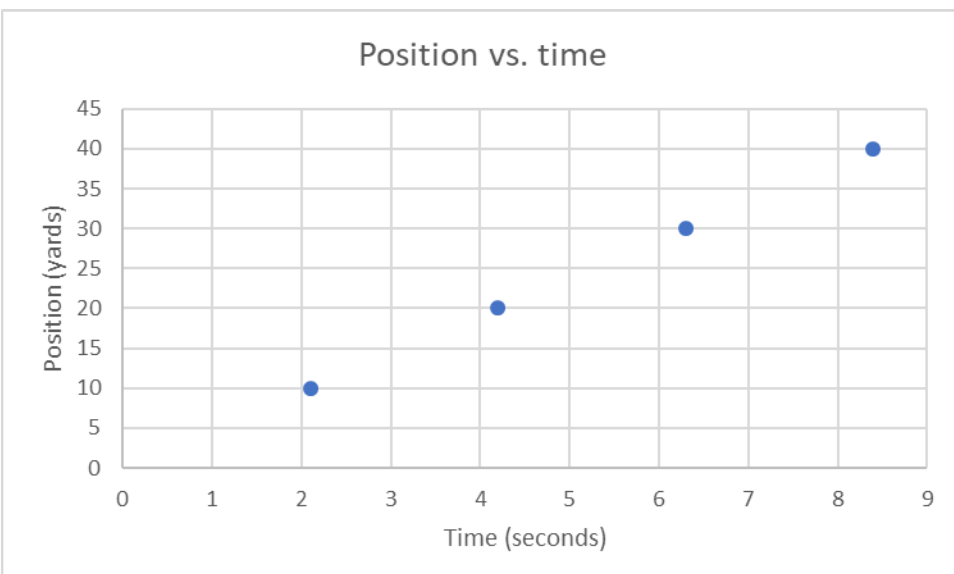
Graphing position versus time

One summer evening, you are watching a soccer match being played on a high school football field and, at some time during the second half, you notice substitute players begin to warm-up by jogging up and down the full length of the sideline.

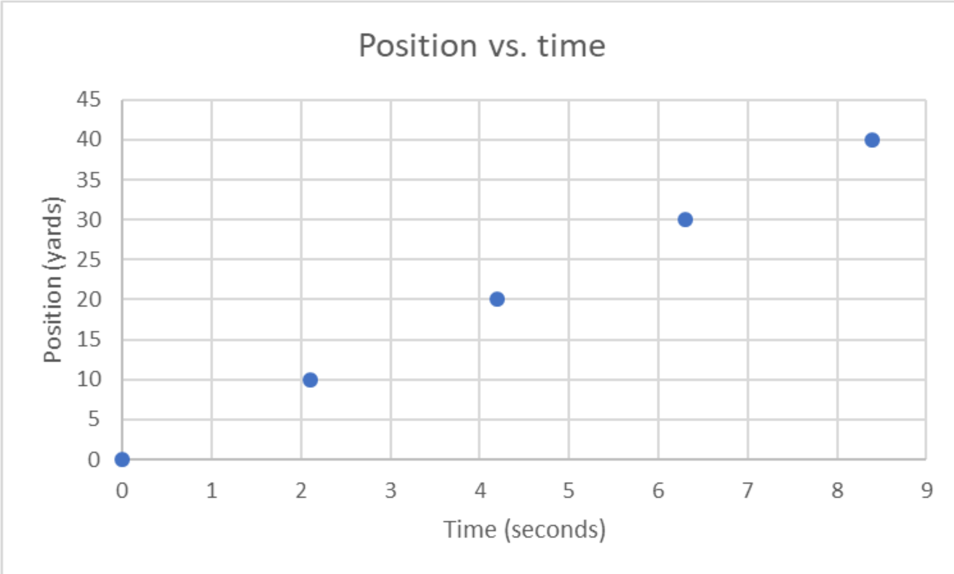
Taking-out your phone and opening the stopwatch app, you press start just as one of the players crosses the zero-yard mark of the football end zone, jogging left to right. You then press the lap button at the 10-yard mark, 20-yard mark, 30-yard mark, and 40-yard mark, collecting the following data:

Time (seconds)	Position (yards)
2.1	10
4.2	20
6.3	30
8.4	40

One way to represent these data in a way that is visually-intuitive is to make a scatter-plot. Even though here the measured time is the dependent variable, we will follow an overriding convention and place it upon the x-axis:

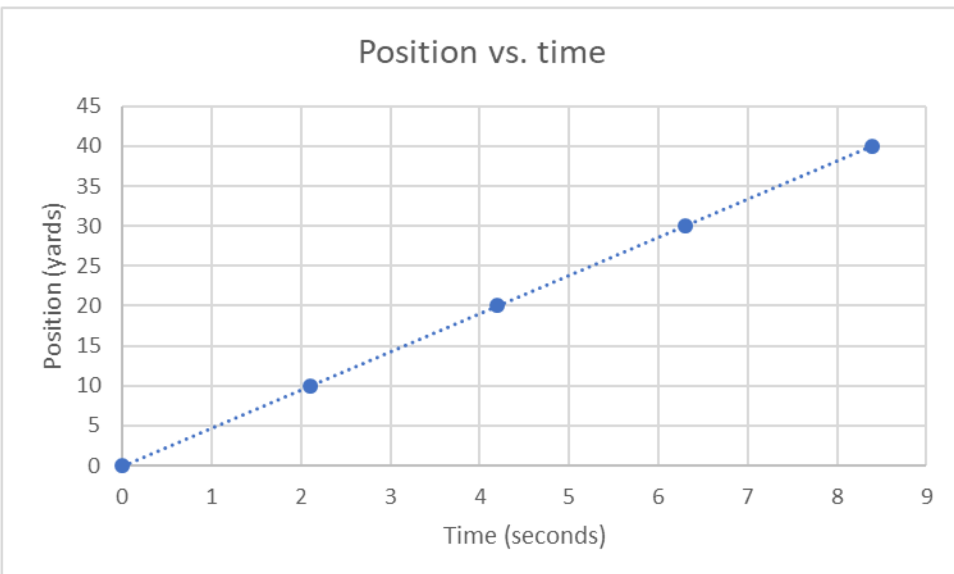


It's also reasonable to add a point at the origin, because we know you began the stopwatch at time zero when the runner had position zero:



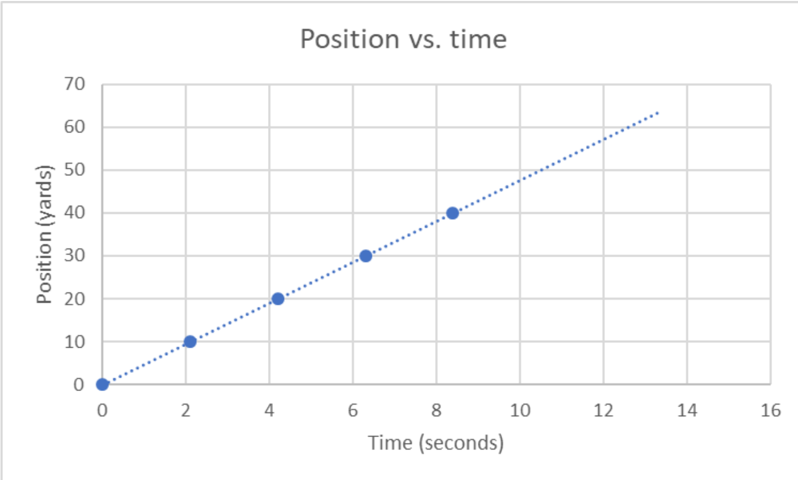
Of course, these measurements have a certain amount of imprecision, but we will leave that concern for a later time.

Also, we don't really know what the runner was doing between these times and positions, but if we want a more complete model of his motion (more than just a series of points), we will need to make an assumption. For simplicity (also because the runner appeared to be moving smoothly), we will suppose that the motion was perfectly gradual, producing a linear regression line:



This graph provides a good sense of where the runner was at various times, at least if the assumption mentioned above is true. We can now approximate that at five seconds of time, the runner had a position of about twenty-four yards. This is an interpolation, an approximation *between* the poles of the lowest and highest data points.

And if we extend the regression line beyond the last data point



we can approximate that the runner reached midfield (50 yards) at about 10.5 seconds, if he kept running at the same rate. This is an extrapolation, an approximation *outside* the poles of the lowest and highest data points.

Furthermore, now that we've modelled the motion with a single line, we can concisely describe it with a mathematical function. We take the standard form of a line:

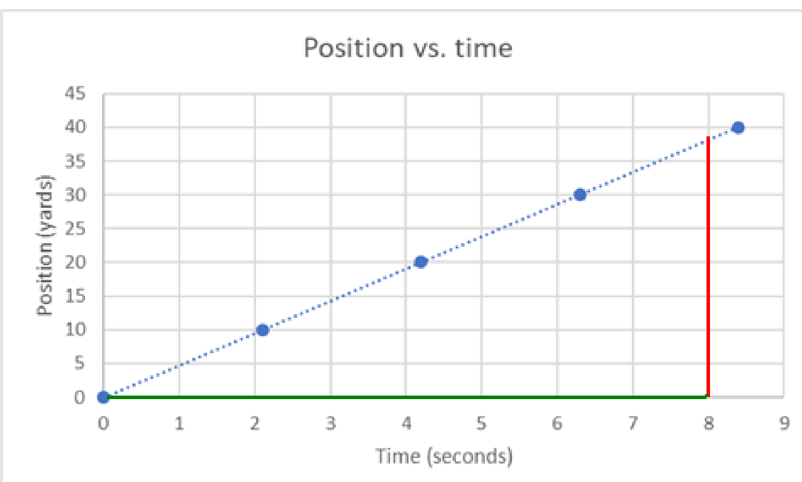
$$y = mx + b$$

and make some substitutions.

Take the general variable *y* and replace it with the concept along the y-axis, *position*.

Take the general variable *x* and replace it with the concept along the x-axis, *time*.

To find *m*, the slope, we can turn the graph into a large right-triangle:



And calculate rise over run or $\frac{\Delta y}{\Delta x}$ or $\frac{change\ in\ position}{change\ in\ time}$ which is about $\frac{38\ yards}{8\ seconds}$ or

4.75 yards/second.

The variable *b* is the y-intercept (where our regression line crosses the y-axis) and here, we can see that is 0 yards.

Altogether the function is now written as:

position = (4.75 yards/second)·time + 0 yards

This can be more compact if we drop the units and the zero-valued y-intercept and replace position and time with standard variables. For position, a standard variable is *s* and for time, the standard variable is *t*.

Altogether then: $s = 4.75 \cdot t$

Read as: *the position of the runner equals 4.75 multiplied by the time on the stopwatch*

We can also use this equation for interpolation and extrapolation. For example, where was the runner at 6.4 seconds? According to this model:

s = (4.75)(6.4) = 30.4 yards.

And how long did it take him to get to 45 yards?

45 = (4.75)(t) so t = 9.47 seconds.

Lastly, the slope of a position versus time graph is so common and useful, it's given its own name, *velocity*.

This is important:

By definition, velocity is the slope of a position versus time graph.