AP Physics 1 – Summer Tutorial 7

Translating functions

The chart of kinematics concepts looks like this:

Position (s)

Velocity (v)

Acceleration (a)

Suppose you have a position function and you would like to find the related velocity function. That requires you to climb down the ladder, so you *differentiate*. How do you differentiate a function? For the ones we'll use here, you follow a two-step process:

1. Multiply the independent term by the exponent
2. Subtract one from the exponent

Here's an example:

A car moves along the ground according to the position function: $s = 10 \cdot t^3$. What is the velocity function?

Step 1: Multiply the independent term by the exponent: $10 \cdot t^3$ becomes $3 \cdot 10 \cdot t^3$.

Visually, you can just imagine swinging the exponent of 3 down in front of the whole $10 \cdot t^3$.

Step 2: Subtract one from the exponent: $3 \cdot 10 \cdot t^3$ becomes $3 \cdot 10 \cdot t^2$

A little simplification gives us the answer, $v = 30t^2$

That's it. Swing the exponent down in front, then take one away from the exponent.

Try this one:

For the position function, $s = -4 \cdot t^2$. What is the velocity function?

Step 1: Multiply the independent term by the exponent: $2 \cdot (-4) \cdot t^2$.

Step 2: Subtract one from the exponent: $2 \cdot (-4) \cdot t^1$

Simplified, this is $v = -8t$

Polynomials aren't that much more challenging:

A car moves along the ground according to the velocity function: $v = 5t^3 - 2t^2$. What is the acceleration function?

Just take the polynomial terms one-at-a-time.

$5 \cdot t^3$ becomes $15 \cdot t^2$

$2t^2$ becomes $4t$

So the acceleration equation is $a = 15t^2 - 4t$

Here are a few other examples:

Given function	Given function, rewritten	When differentiated	Then simplified
s = 16t	$s = 16t^1$	$v = 1 \cdot 16 \cdot t^0$	v = 16
s = 25	$s = 25 \cdot t^0$	$v = 0 \cdot 25 \cdot t^{-1}$	v = 0
$s = \sqrt{t}$	$s = t^{1/2}$	$v = \frac{1}{2} \cdot t^{-1/2}$	$v = \frac{1}{2\sqrt{t}}$
$s = \frac{1}{t}$	$s = t^{-1}$	$v = -1 \cdot t^{-2}$	$v = \frac{-1}{t^2}$

If you've forgotten some math, the last two might be tricky, but don't worry about that. The first two should make sense, conceptually.

If s = 16t, this means that the position increases by 16 for every 1 unit of time, but that's just what a velocity of v = 16 also means.

And if s = 25, the position stays at 25 regardless of time, which agrees with a velocity of zero.

Try these for practice; the answers are on the next page.

1. Differentiate $s = t^8 + t^5 - t^2$

2. Differentiate $s = 12t^4 - 5t^2 + 12$

3. Differentiate $s = 0.25t^4 + t^2 – 6$

4. Given the equation for position, $s = 12t^2 + t – 5$, what is the equation for velocity?

5. Given the equation for velocity, $v = -4t^2 + 12t + 8$, what is the equation for acceleration?

6. Suppose $s = 4t^3 + 2t + 5$. How can the acceleration be determined?

Answers:

1. $v = 8t^7 + 5t^4 – 2t$

2. $v = 48t^3 – 10t$

3. $v = t^3 + 2t$

4. $v = 24t + 1$

5. $a = -8t + 12$

6. $v = 12t^2 + 2$ then differentiate again for $a = 24t$

Suppose you know the velocity function and would like to translate that into a position function. There should be a reverse process to "climb the ladder" and there is. This process is called *integration*.

Position (s)

Velocity (v)

Acceleration (a)

All we really need to do is reverse the two steps of differentiation. So the steps for *integration* are:

Step 1: Add one to the exponent

Step 2: Divide by the new exponent

Here's an example:

A particle moves away from an electrostatic generator with the velocity function:

$v = 12t^3$. What is the position function?

Step 1: Add one to the exponent $12t^3$ becomes $12t^4$

Step 2: Divide by the new exponent $12t^4$ becomes $\frac{12t^4}{4}$

Simplified, this is: $s = 3t^4$

Technically, what we really get from the integration is just information about the *change* in position, so we should write the result at:

$$\Delta s = 3t^4$$

Think of it this way: If I tell you that someone is driving 60 mph east (a known velocity function), you have some idea of what their change in position is over the next two hours, but you have no idea what their actual position is on the surface of the planet.

Another example:

If $v = 36t^3 - 10t + 15$, what is the equation for change in position?

First, note that $10t$ is the same as $10t^1$ and 15 is the same as $15t^0$.

$v = 36t^3 - 10t^1 + 15t^0$

$$\Delta s = \frac{36t^4}{4} - \frac{10t^2}{2} + \frac{15t^1}{1}$$

$$\Delta s = 9t^4 - 5t^2 + 15t$$

Practice (answers are on the next page)

1. If $v = 12t^2 - 6t + 4$, what is the equation for change in position?
2. If $a = 4t + 8$, what is the equation for change in velocity?
3. If $v = 6t^2 + 10t$, what is the equation for change in position?
4. If $a = 24t^2 - 3$, what is the equation for change in velocity?
5. If $v = 15t^2 - 12$, what is the equation for change in position?
6. If $a = 9t^2 + 8t - 2$, what is the equation for change in velocity?

Answers:

1. $\Delta s = 4t^3 - 3t^2 + 4t$

2. $\Delta v = 2t^2 + 8t$

3. $\Delta s = 2t^3 + 5t^2$

4. $\Delta v = 8t^3 - 3t$

5. $\Delta s = 5t^3 - 12t$

6. $\Delta v = 3t^3 + 4t^2 - 2t$