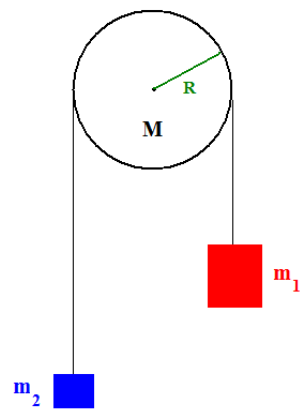
Atwood's machine



In the diagram above, there are two blocks attached to either end of a cord. The cord has negligible mass and is strung around a solid disk pulley with mass M and radius R. What is the linear acceleration of each block and what is the angular acceleration of the pulley?

There are several ways to solve this and I will begin with the easiest.

You can imagine making the cord shorter and shorter until the two blocks are essentially stuck to the pulley. Then the total rotational inertia of the system is just the rotational inertia of the two blocks plus the rotational inertia of the pulley:

$I_{system} = m_2R^2 + m_1R^2 + \frac{1}{2}MR^2 = (m_2 + m_1 + \frac{1}{2}M)\cdot R^2$

Taking clockwise to be negative, the net torque is then $m_2 \cdot g \cdot R - m_1 \cdot g \cdot R$

$\Sigma\tau = I\cdot\alpha$ therefore

$m_2 \cdot g\cdot R - m_1 \cdot g\cdot R = [(m_2 + m_1 + \frac{1}{2}M)\cdot R^2]\cdot\alpha$

$$\alpha = \frac{g(m_2 - m_1)}{(m_2 + m_1 + \frac{1}{2}M)\cdot R}$$

And if $a = \alpha\cdot R$, then $a = \dfrac{g(m_2 - m_1)}{(m_2 + m_1 + \frac{1}{2}M)}$

A second way to solve this is with conservation of energy. Let's suppose the system begins at rest and m_1 falls a distance *h*. Taking the pulley, blocks, and Earth as the system, all forces are internal, so no work is done on the system.

$\Delta E = 0$

$\Delta U_g + \Delta K = 0$

m_1 is falling, so that change in potential energy is negative
m_2 is rising, so that change in potential energy is positive

$(m_2 \cdot g \cdot h) + (-m_1 \cdot g \cdot h) + (\frac{1}{2} \cdot m_1 \cdot v^2) + (\frac{1}{2} \cdot m_2 \cdot v^2) + (\frac{1}{2} \cdot I \cdot \omega^2) = 0$

where $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$

$(\frac{1}{2} \cdot m_1 + \frac{1}{2} \cdot m_2 + \frac{1}{4} \cdot M) \cdot v^2 = g \cdot h \cdot (m_1 - m_2)$

We know the forces are independent of the velocities, so the acceleration must be constant, meaning we can use the equation $v_f^2 = v_i^2 + 2(a)(\Delta s)$

$v^2 = 2(a)(h)$ which I can put into the equation above for v^2

$(\frac{1}{2} \cdot m_1 + \frac{1}{2} \cdot m_2 + \frac{1}{4} \cdot M) \cdot 2(a)(h) = g \cdot h \cdot (m_1 - m_2)$

Rearranging for acceleration yields $a = \frac{g(m_2 - m_1)}{(m_2 + m_1 + \frac{1}{2}M)}$ which is the same result as before

---

The final process is the most difficult and requires careful assignment of signs. We will look at the three components of the system individually, again taking clockwise as negative.

Using $F = m \cdot a$, for the block on the left, we have

$+m_2 \cdot g - T_2 = m_2 \cdot a$ where T_2 is the magnitude of the tension of the cord above block 2

For the block on the right,

$-m_1 \cdot g + T_1 = m_1 \cdot a$ where T_1 is the magnitude of the tension of the cord above block 1

The two tensions must be different; if they were the same, the pulley would not experience a net torque and so would not spin.

For the pulley, we have

$\Sigma\tau = I\cdot\alpha$ where $\alpha = \frac{a}{R}$

$-T_1\cdot R + T_2\cdot R = \frac{1}{2}M\cdot R^2\cdot\frac{a}{R} = \frac{1}{2}M\cdot R\cdot a$ or $-T_1 + T_2 = \frac{1}{2}M\cdot a$

Rearranging the top two equations for T_1 and T_2 and inserting them into the equation above is then:

$-(m_1\cdot a + m_1\cdot g) + (m_2\cdot g - m_2\cdot a) = \frac{1}{2}M\cdot a$

Solving for acceleration yields:

$$a = \frac{g(m_2 - m_1)}{(m_2 + m_1 + \frac{1}{2}M)}$$

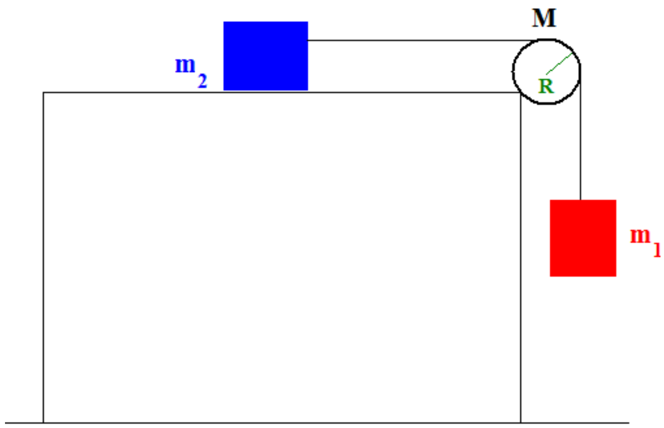
Answer Webassign Question 1

Answer Webassign Question 2

Answer Webassign Question 3

If the situation is instead a modified Atwood's machine, simply remove m_2 from the numerator.

Answer Webassign Question 4



If this system has friction, simply add $\mu\cdot m_2\cdot g$ to the numerator

Answer Webassign Question 5