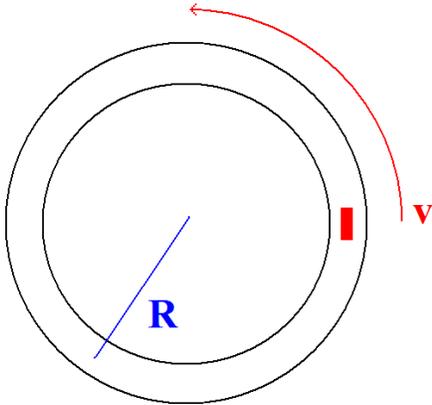
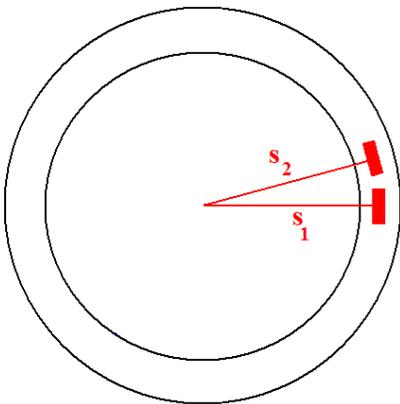


Centripetal acceleration

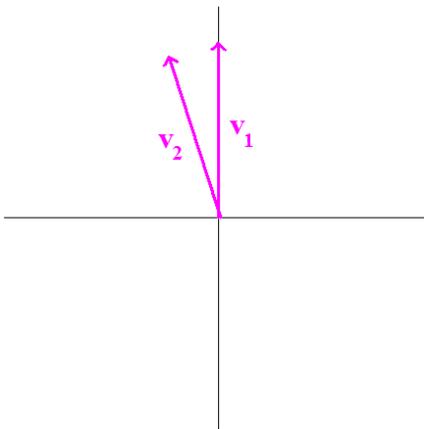
Suppose you had a car moving at a constant speed, v , counter-clockwise around a circle with radius, R .



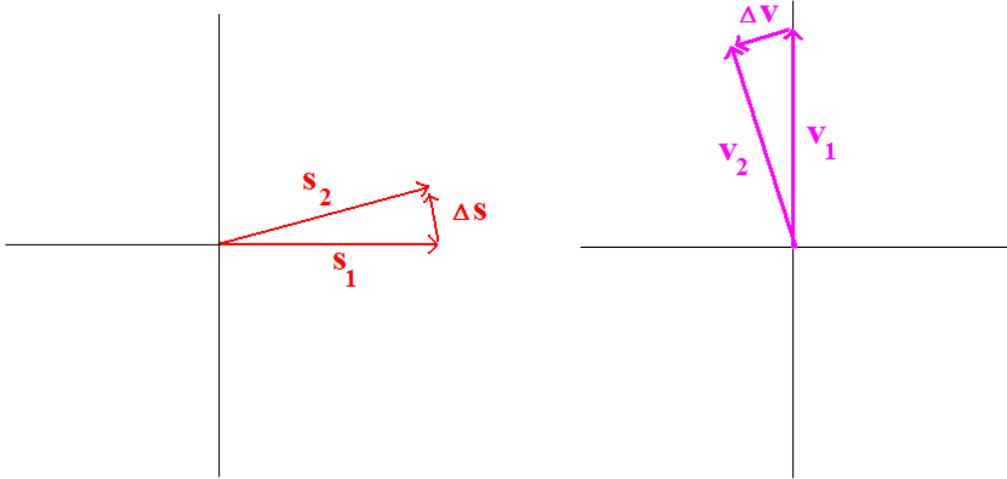
At one instant in time, it may be at position 1 relative to the center of the circle. And a very short time later, it will be at position 2.



And even though the speed is constant, the velocity is changing because the direction of motion is changing. This can be seen if I draw a vector diagram of the velocity at position 1 and the velocity at position 2.



Now I'll redraw the vector diagrams for position and velocity and add vectors for the changes in position and velocity.



Because v_1 is perpendicular to s_1 and v_2 is perpendicular to s_2 , these are similar triangles. In terms of the magnitudes of the sides, this allows the proportion:

$$\frac{\Delta s}{s_1} = \frac{\Delta v}{v_1}$$

Again, in terms of magnitudes, $s_1 = R$ and $v_1 = v$. Also, $\Delta s = v \cdot \Delta t$

So now, the equation becomes

$$\frac{v \cdot \Delta t}{R} = \frac{\Delta v}{v} \quad \text{or} \quad \frac{v^2}{R} = \frac{\Delta v}{\Delta t}$$

By definition, $\frac{\Delta v}{\Delta t}$ is an acceleration, so this yields the final result:

$$a_c = \frac{v^2}{R}$$

Answer Webassign Question 1

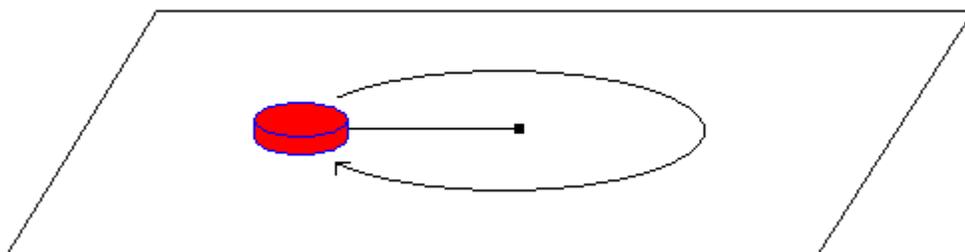
The small subscript c specifies that this is a *centripetal* acceleration. It is called this because centripetal means “center-seeking”. If you look back at the velocity vector diagram, the change in velocity vector points down and to the left, anti-parallel to the s_2 vector. The s_2 vector is pointing away from the center of the circle, so the Δv vector is pointing towards the center of the circle. If that is true and $a_c = \frac{\Delta v}{\Delta t}$, then a_c is pointing towards the center of the circle as well.

By Newton's second law, any acceleration must come from a force or combination of forces. This is called the centripetal force. If $F = m \cdot a$ and $a_c = \frac{v^2}{R}$, then $F_C = m \cdot a_c$ or $F_c = m \cdot \frac{v^2}{R}$.

Answer Webassign Question 2

In the case of the car, the centripetal force is coming from the force of friction between the road and the tires. If you take that force away (with the car hitting a patch of ice, for instance), the car will no longer circulate and simply slide off the track with a constant velocity (constant speed, constant direction).

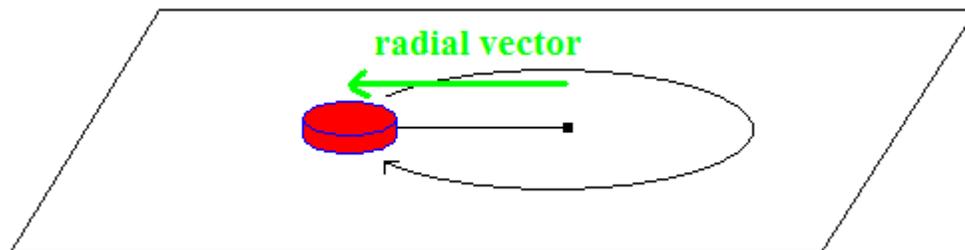
We can use these equations with a couple of examples. Suppose you had a 2kg puck tied to a 70cm string and the puck was sliding around in a circle on an air hockey table at a speed of 4m/s. What would be the tension in the string?



The centripetal acceleration is easy to determine. $a_c = \frac{v^2}{R} = \frac{(4m/s)^2}{0.7m} = 22.86m/s^2$.

The centripetal force is simply mass times acceleration, $(2kg)(22.86m/s^2) = 45.72N$.

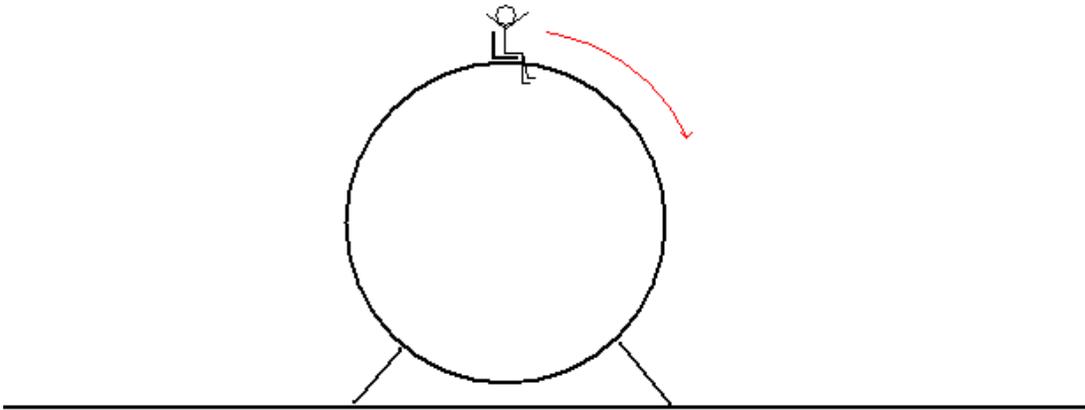
Where is this centripetal force coming from physically? In other words, what forces act on the puck parallel or antiparallel to the radial vector at some instant in time?



The force of gravity and the force of the table and perpendicular to this radial vector, so they can't contribute. The only force that can is the tension in the cord, so the tension must provide the entire centripetal force, 45.72N.

Answer Webassign Question 3

Here's a slightly more complex example. Suppose you have a mass of 70kg and ride a Ferris wheel with a radius of 10m so that you are moving at 5m/s. What forces act on you when at the top of the ride?



Now, if the ride was stationary, this would be easy. The force of gravity would be -686N and the normal force from the seat will be +686N. But because you are moving in a circle, you are accelerating with centripetal acceleration and that will change the normal force of the seat.

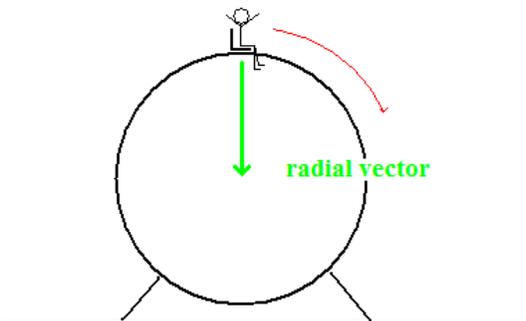
First, just calculate the centripetal acceleration and centripetal force.

$$a_c = \frac{v^2}{R} = \frac{(5\text{m/s})^2}{10\text{m}} = 2.5\text{m/s}^2$$

But we need to be careful here. Centripetal means towards the center and when you are at the top of the Ferris wheel, towards the center points down. So the actual centripetal acceleration is -2.5m/s^2 .

This times your mass is the centripetal force, -175N.

Now, what forces are parallel or antiparallel to the radial vector that could contribute to this centripetal force?



They are gravity and the seat. So conceptually, the force of gravity plus the force of the seat add to provide the centripetal force.

$$F_G + F_{\text{seat}} = F_C$$

Filling that in becomes $(-686\text{N}) + F_{\text{seat}} = -175\text{N}$ and solving for F_{seat} yields 511N. Here, the seat is pushing up less than it would if you were sitting on the ride stationary, which is why you feel slightly thrown up into the air on rides like this.

If you were asked how fast the Ferris wheel would need to spin so that you “fly off” the seat, you would simply set the force of the seat to zero and solve backwards to find the velocity.

Answer Webassign Question 4

Answer Webassign Question 5

To review:

Centripetal acceleration follows the equation $a_c = \frac{v^2}{R}$

The sum of all forces acting along the radius to cause this acceleration is the centripetal force

$$F_c = m \cdot \frac{v^2}{R}$$

“Centripetal” means towards the center. Both the centripetal acceleration and centripetal force vectors point towards the center of the circle in which the object moves.