

Conservation of energy

Just to recapitulate the types of energy known so far:

$$K = \frac{1}{2}mv^2$$

$$U_{\text{gravitational}} = m \cdot g \cdot h$$

$$U_{\text{spring}} = \frac{1}{2}k \cdot x^2$$

Answer Webassign Question 1

The equation we will use for conservation of energy is this:

$$\Delta E = W + Q$$

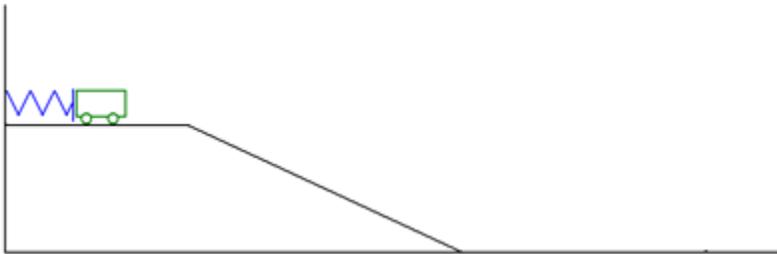
ΔE is the total change in energy of the system, which could be in the form of thermal energy, chemical energy, gravitational potential energy, spring potential energy, and kinetic energy. There are other ways to categorize energies, but these are the ones we will use.

W is the work input to the system

Q is heat flowing into the system (+) or out of the system (-). This is useful in thermodynamic processes like engines and refrigerators but which are not part of this class, so Q will most likely always be zero.

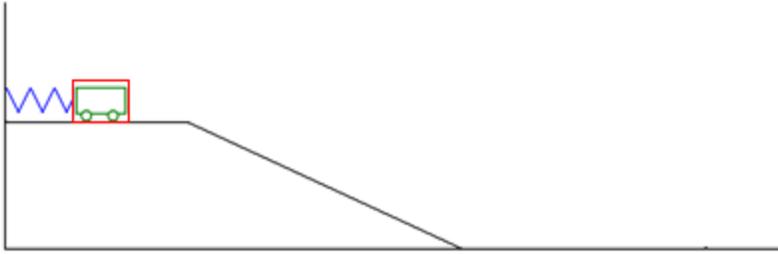
Answer Webassign Question 2

So let's see how this can be applied to a certain situation.



A cart begins at the top of a plateau, pushed into a spring a certain distance. The cart begins at rest, but when the spring is released, the cart rolls down the incline and along the floor.

How this situation is analyzed in terms of work and energy depends upon what is chosen as the system and what is chosen as the environment. Let's start by just taking the cart itself as the system.



Does the system begin with kinetic energy? No, nothing is moving at first.

Does it begin with gravitational potential energy? No. Even though the cart is above the ground, we have chosen the cart alone as the system and there is no cart/Earth separation in this system.

Does it begin with spring potential energy? No. Again, the compressed spring is not part of the system as defined.

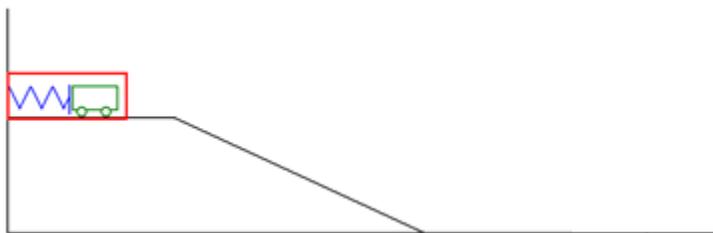
What does work on the system? The spring does work because it is a force applied to the system from outside the system which occurs over a displacement of the cart. Likewise for the force of gravity as the cart descends the ramp.

Lastly, the cart ends with kinetic energy as it rolls along the floor.

So $\Delta E = W + Q$ would be written as $\frac{1}{2}mv^2 = W_{\text{spring}} + W_{\text{gravity}}$

Answer Webassign Question 3

Let's use the same situation, but define the system as the cart and spring together.



To begin:

$$K = 0$$

$$U_{\text{gravitational}} = 0$$

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

When the spring is fired, however, it does no work on the system because it is internal to the system. In the firing, there is a change in spring potential energy of $-\frac{1}{2}kx^2$.

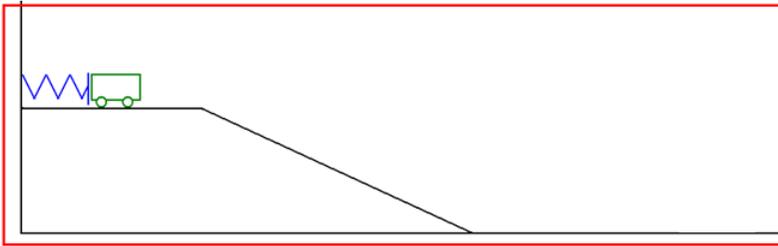
In the descent, gravity does work on the cart.

Altogether, we have:

$$-\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = W_{\text{gravity}}$$

Answer Webassign Question 4

Suppose you focus on everything, including the Earth.



Now we do begin with spring potential energy and gravitational potential energy, because the compressed spring and Earth/cart separation are included in our system. The spring force and gravitational force are internal to the system, so they input no work.

Altogether, we have

$$-\frac{1}{2}kx^2 + (-mgh) + \frac{1}{2}mv^2 = 0$$

Answer Webassign Question 5

There's one further technical point to be made. Suppose we take as our system a block sitting on the floor at rest. When the block is treated as an *object*, we can say that it has zero energy. Conceptualizing the block as an object assumes it has no internal structure.

But if we zoomed in to the particles which make-up the block, we are then conceptualizing the block as a *system* of particles. Now the block does have an internal structure and it can have kinetic energies of the particles vibrating in place and potential energies within the molecular and atomic bonds. This sum of kinetic and potential energies is the *internal energy* of the block.

When a situation like the one described above is analyzed in such a way that the object is the system, there is no possibility of potential and internal energies, so the conservation of energy equation reduces to $\Delta K = \Sigma W$, sometimes called the work-energy theorem.

At the other extreme, one can take the entire universe to be the system, so that external work and heat flowing into the system are impossible and the conservation of energy equation reduces to $\Delta E = 0$.