Conservation of momentum

In the notes on center of mass, we saw:

$$(\Sigma F_{ext})(\Delta t) = (M)(a_{cm})(\Delta t)$$

$$(\Sigma F_{ext})(\Delta t) = (M)(\Delta v_{cm}) = m_1 \cdot \Delta v_1 + m_2 \cdot \Delta v_2 + m_3 \cdot \Delta v_3 + \cdots$$

In the special case of a net external force of zero, we can write it as:

$$0 = \Delta m_1 v_1 + \Delta m_2 v_2 + \Delta m_3 v_3 + \cdots$$

Stated as the law of conservation of momentum:

If the net external force on a system is zero, then there is no overall change in the momentum of the system. What momentum you begin with is the momentum you end with.

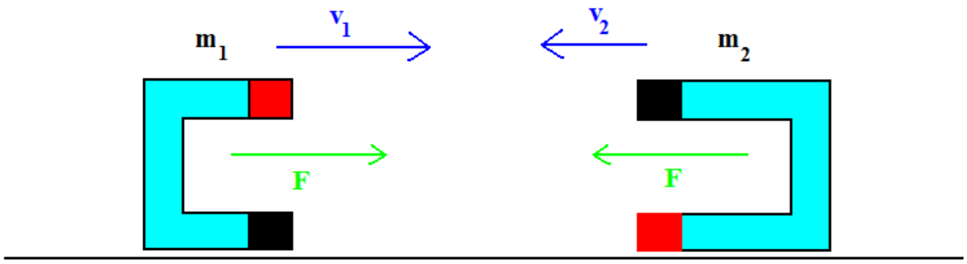
If $\Sigma F_{ext} = 0$ then $\Sigma p_i = \Sigma p_f$

Answer Webassign Question 1

Clearly, if there is a net external force acting on a system, it can change momentum. Kicking a ball is a simple example.

Here's a illustration. Suppose you have two very large magnets sitting on a frictionless surface, each beginning at rest.

In this orientation, the two magnets will be attracted to each other, each experiencing the same magnitude of force from the attraction by Newton's third law. Therefore, they will begin sliding together.



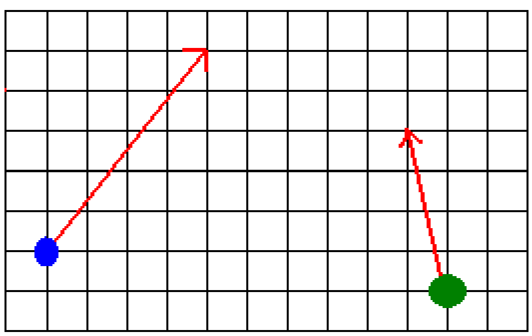
Let's say we look at the amount they both slide during a time *Δt*. If we call *F* just the magnitude of the force, the left-hand magnet will experience an impulse of $+F \cdot \Delta t$. The right-hand magnet will experience an impulse of $-F \cdot \Delta t$.

Impulse equals change in momentum, so the left-hand magnet will change momentum by $+F \cdot \Delta t$ the right-hand magnet with change momentum by $-F \cdot \Delta t$.

Therefore, the total change in momentum for the system is zero.

As with the concepts of impulse and momentum, the derivation above holds just as true in two and three dimensions. It is a vector equation.

$\Sigma \mathbf{p_i} = \Sigma \mathbf{p_f}$ if $\Sigma \mathbf{F}_{external} = 0$



If the diagram above has units of kg·$\frac{m}{s}$, the little blue puck would have a momentum of < 4, 5> kg·$\frac{m}{s}$

the little green puck would have a momentum of <-1, 4> kg·$\frac{m}{s}$,

making the total momentum of the system < 3, 9 > kg·$\frac{m}{s}$.

The two pucks may collide and stick together or collide and bounce off each other, but whatever happens, the total momentum of this closed system will remain < 3, 9 > kg·$\frac{m}{s}$.

Answer Webassign Question 2

A very common application for conservation of momentum involves the collision between two or more objects in motion.

Take, for instance, two small carts on a track rolling towards each other. Because the net external force is zero, $\Sigma p_i = \Sigma p_f$.

There are three possible results for the collision.

1. If the collision is *perfectly inelastic*, then the two carts will stick together and move at some common final velocity. By conservation of momentum:

$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = (m_1 + m_2) \cdot v_f$

Answer Webassign Question 3

2. If the collision is *inelastic*, then the cars may not stick together. They may bounce off each other, but some kinetic energy will be lost in the collision.

$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot v_{1f} + m_2 \cdot v_{2f}$ and $K_f < K_i$

Answer Webassign Question 4

Answer Webassign Question 5

3. If the collision is *elastic*, then the cars will bounce off each other and no kinetic energy is lost in the collision. Therefore, there are two conservation equations.

$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot v_{1f} + m_2 \cdot v_{2f}$

$\frac{1}{2} m_1 \cdot v_{1i}^2 + \frac{1}{2} m_2 \cdot v_{2i}^2 = \frac{1}{2} m_1 \cdot v_{1f}^2 + \frac{1}{2} m_2 \cdot v_{2f}^2$

If there is a net external force, then neither momentum nor kinetic energy will be conserved. For instance, if these were blocks sliding along a rough surface instead of carts rolling.

To review:

Momentum is mass times velocity, $\mathbf{p} = m \cdot \mathbf{v}$

If the net external force acting on a system is zero, the total momentum of the system remains constant.

Symbolically, if $\Sigma \mathbf{F}_{external} = 0$ $\quad \Sigma \mathbf{p_i} = \Sigma \mathbf{p_f}$

Collisions can be perfectly inelastic, inelastic, or elastic.