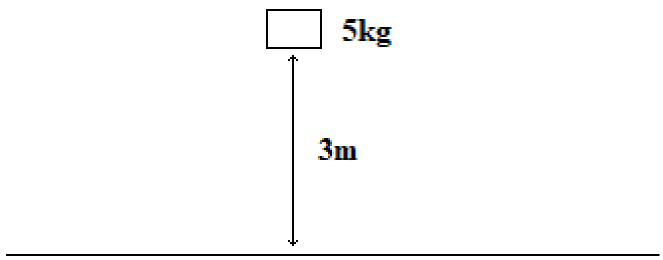
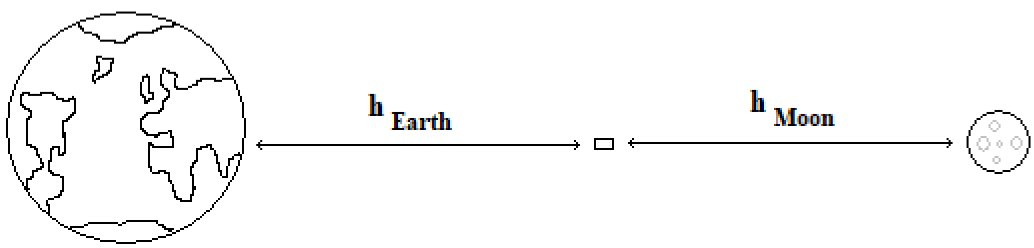
# Gravitational Potential Energy – Astronomic Reference Frames

By now, you are familiar with the idea of gravitational potential energy, $U_G = mgh$. For example, with a 5kg cinderblock held 3m above the ground, the Earth/cinderblock system has a gravitational potential energy of $(5kg)(10m/s^2)(3m) = 150J$.



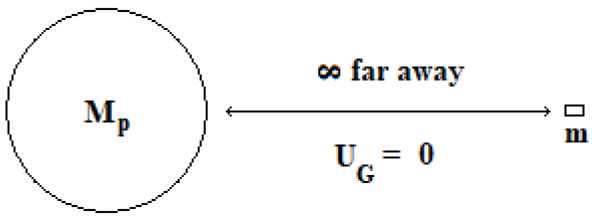

Here, it is convenient to define the surface of the Earth (say, sea-level) as a height of zero. But suppose we keep lifting the cinderblock higher and higher away from the Earth until it is exactly midway between the Earth and the Earth's moon.




Should we use the surface of the Earth as a reference for zero potential energy, or should we use the surface of the moon? There's really no way to correctly answer that question without being provincial. Instead, we need to find some position which is equally distant from all surfaces, so that all surfaces have equal standing in our definition of gravitational potential energy.

This might sound like an unsolvable puzzle, "What point in space is equally distant from everywhere?", but the solution is the concept of infinity. A point in space infinitely far-away is infinitely (and equally) far from all points in space.

So let's imagine a simple universe with only one planet which has a mass $M_p$ and a radius $R_p$. If we have a cinderblock of mass *m* infinitely far away, how much potential energy should the cinderblock/planet have? We have no reason to prefer one number over another, which leaves us with two choices, zero and infinity. If we choose an infinite potential energy at an infinite distance, nothing is going to be solvable when the situation changes, so that leaves us with defining the potential energy at an infinite distance as zero.



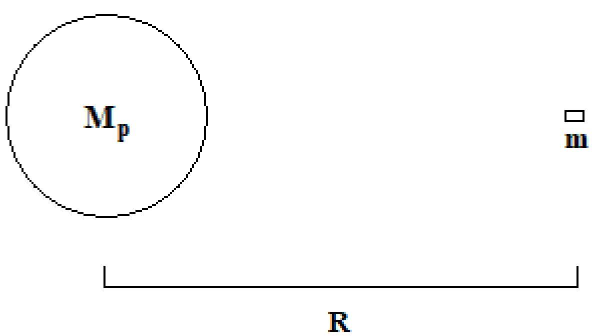

If the cinderblock moves closer to the planet, the force of gravity on the cinderblock is *towards* the planet and the displacement of the cinderblock is *towards* the planet, making the work done by the planet's gravitational field on the cinderblock positive using $W = F \cdot \Delta s \cdot (\cos 0^o)$.

If $W = -\Delta U$ and the work done by gravity is positive, then the change in potential energy is negative. This means that the potential energy of the cinderblock/planet system will always be negative and will be more-and-more negative the closer-and-closer they get to each other.

Using calculus, one can derive the equation for the potential energy of the cinderblock/planet system as:

$$U_G = \frac{-G \cdot m_1 m_2}{R}$$

where G is the same gravitational constant, $6.67 \times 10^{-11}\ N \cdot m^2/kg^2$
$m_1$ is the planet mass
$m_2$ is the cinderblock mass
R is the distance between the center of the planet and the center of the cinderblock




This equation is very similar to the equation for the universal force of gravity, except it is always negative and the denominator is R rather than $R^2$. You can also see that the equation produces a negative number for any pair of masses at any distance (the one exception being zero potential energy when $R = \infty$).