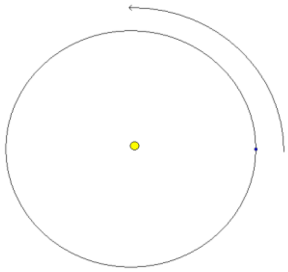
Gravity and satellites

Here is a simple picture, not at all to-scale, of the Earth orbiting the sun.



Because the Earth is in what is close to a perfect circle, it experiences a centripetal acceleration which is caused by a centripetal force. In this case, the centripetal force is the force of gravity that the sun exerts on the Earth.

From astronomic data, Isaac Newton determined the equation for this force as follows:

$$F_G = \frac{G \cdot m_1 \cdot m_2}{R^2}$$

F_G is the force of gravity

G is the universal gravitational constant, which has a value of $6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

m_1 is the gravitational mass of the first interacting object, here the mass of the sun
m_2 is the gravitational mass of the second interacting object, here the mass of the Earth
R is the distance between the center of the sun and the center of the Earth

Answer Webassign Question 1

Answer Webassign Question 2

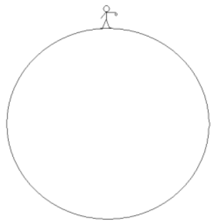
Gravitational mass is the name given to the mass responsible for this force. Inertial mass is the name given to the property of matter than makes it resist acceleration. The two are different conceptually, but the same numerically. In other words, when you drop an object:

$- m_{gravitational} \cdot g = m_{inertial} \cdot a$ and because all objects experimentally have a freefall acceleration equal to –g

$m_{gravitational} = m_{inertial}$

Technically, every little piece of the sun pulls on every little piece of the Earth, but mathematically this works out the same as if the mass of each was localized at its center. This is why we can use the distance between the centers in the equation. Newton proved this with calculus, but the proof is rather extensive, so I will just include a link on the website.

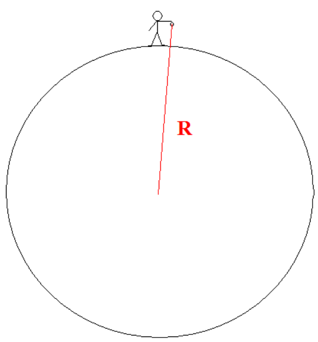
Now suppose you stood at the North Pole and dropped an apple from 2m above the ground. Again, this diagram is not to-scale.



Any two objects with mass will attract each other with this force of gravity, so when you release the apple it will fall under the influence of the pull of the Earth. Filling in the above equation yields:

$$F_G = \frac{G \cdot m_{apple} \cdot m_{Earth}}{R^2}$$

Here R is the distance between the center of the Earth and the center of the apple. Disregarding the 2m which is trivial relative to the radius of the Earth, we can replace R with $R_{Earth,}$ the physical radius of the Earth.



When falling, the net force on the apple is simply the force of gravity, or $m_{apple} \cdot g$. So we can replace F_G with $m_{apple} \cdot g$ and solve for g.

$$m_{apple} \cdot g = \frac{G \cdot m_{apple} \cdot m_{Earth}}{{R_{Earth}}^2}$$ and dividing both sides by m_{apple} leaves:

$$g = \frac{G \cdot m_{Earth}}{{R_{Earth}}^2}$$

You can check that using 6.67 x $10^{-11}\ \frac{N \cdot m^2}{kg^2}$, the Earth's mass of 5.972 x 10^{24}kg, and Earth's radius of 6.371 x 10^6m produces a number you've seen before.

This equation works for any planet, moon, or star. In short, the acceleration due to gravity on the surface of such an astronomic body is:

$$g = \frac{G \cdot m}{R^2}$$

Answer Webassign Question 3

Now go back to the diagram of the Earth orbiting the sun. We can say that the Earth is a satellite relative to the sun and derive several equations for satellites.

If the force of gravity is a centripetal force, we can replace F_G in the equation with $m_{Earth} \cdot \frac{v^2}{R}$

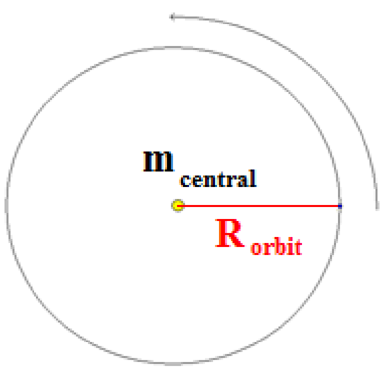
$$m_{Earth} \cdot \frac{v^2}{R} = \frac{G \cdot m_{Earth} \cdot m_{sun}}{R^2} \quad \text{or} \quad v = \sqrt{\frac{G \cdot m_{sun}}{R}}$$

In general, for any satellite, the Earth around the sun, the moon around the Earth, the international space station around the Earth:

$$v_{satellite} = \sqrt{\frac{G \cdot m_{central}}{R_{orbit}}}$$

where $m_{central}$ is the mass of the object around which the satellite moves and R_{orbit} is the distance between the center of the central mass and the center of the satellite.

Answer Webassign Question 4



If the satellite moves in a circle, then it travels one circumference of distance in the time of one orbit, known as the period and symbolized with *T*. Therefore $v_{satellite} = \frac{2\pi \cdot R_{orbit}}{T}$

Putting this into the equation above yields

$$\frac{2\pi \cdot R_{orbit}}{T} = \sqrt{\frac{G \cdot m_{central}}{R_{orbit}}}$$

Rearranging for T yields

$$T = \sqrt{\frac{4\pi^2 \cdot R_{orbit}^3}{G \cdot m_{central}}}$$

Answer Webassign Question 5

To review, any two masses will attract each other with a gravitational force given by the equation:

$$F_G = \frac{G \cdot m_1 \cdot m_2}{R^2}$$

The acceleration due to gravity on the surface of a planet, moon, or star is:

$$g = \frac{G \cdot m}{R^2}$$

And satellites moving in circular orbits follow the equations:

$$V_{satellite} = \sqrt{\frac{G \cdot m_{central}}{R_{orbit}}} \quad \text{and} \quad T = \sqrt{\frac{4\pi^2 \cdot R_{orbit}^3}{G \cdot m_{central}}}$$