Kinematics in one dimension

In classical physics, there are three dimensions in which one can move, the most common convention being the following:

Moving left and right is horizontal motion and is along the x-axis of the Cartesian coordinates, leftward being negative and rightward being positive.

Moving up and down is vertical motion and is along the y-axis, up being positive and down being negative.

Moving into and out of the page is along the z-axis, into the page being negative and out of the page being positive.

However, understanding is easiest when one begins with simple situations and works towards the more complex, so the study of kinematics is best begun with restricting motion to one of these dimensions, for example, a car moving along a straight, level road or a ball simply rising and then falling.

The simplest of these motions is an object with zero velocity. There is not much to say about this. The position is constant, the velocity is zero, and the acceleration is zero. This can be notated as: $s = k$, $v = 0$, $a = 0$. *k* is used to represent some constant number because the word constant begins with a k in German.

The next simplest is motion with constant velocity, where the definition of velocity is change in position divided by the corresponding change in time, or $v = \frac{\Delta s}{\Delta t}$ where $\Delta s = s_f - s_i$ and $\Delta t = t_f - t_i$. By definition, $a = 0$. These can be rearranged into a variety of useful equations:

$s_f = s_i + \Delta s$ read as "final position equals initial position plus change in position:

$\Delta s = v \cdot \Delta t$ "change in position equals velocity multiplied by time span"

$s_f = s_i + v \cdot \Delta t$

$\Delta t = \frac{\Delta s}{v}$

Answer Webassign Question 1

Next is motion with constant acceleration. By definition, $a = \frac{\Delta v}{\Delta t}$ where $\Delta v = v_f - v_i$. These can be rearranged into:

$v_f = v_i + \Delta v$

$\Delta v = a \cdot \Delta t$

$v_f = v_i + a \cdot \Delta t$

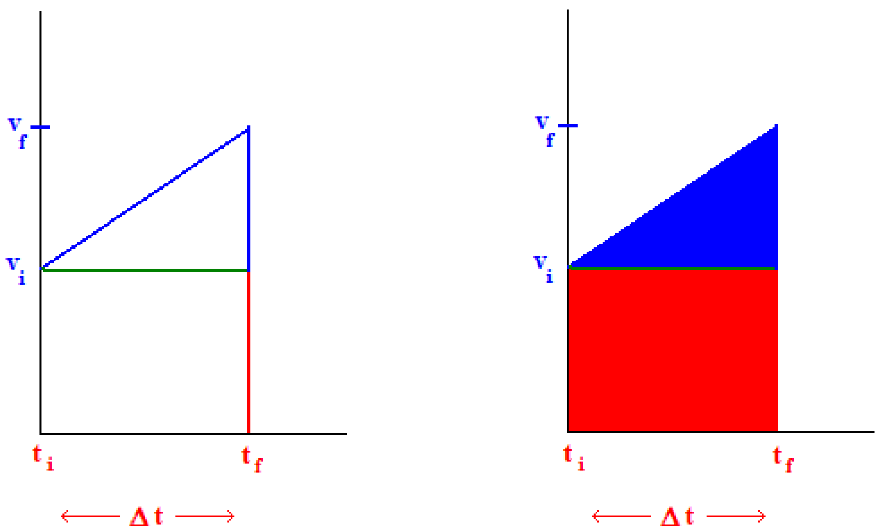
$\Delta t = \frac{\Delta v}{a}$

Answer Webassign Question 2

These can then be coupled to the definition of average velocity, $\bar{v} = \frac{\Delta s}{\Delta t}$. This looks like the same equation for velocity defined earlier, but $v = \frac{\Delta s}{\Delta t}$ is only true when the velocity is constant and $\bar{v} = \frac{\Delta s}{\Delta t}$ is always true. This is a technical point, so if it's not perfectly clear, there's not much to worry about.

Anyway, with this idea of average velocity, several further equations can be derived. You're not responsible for memorizing the derivations, but the results are useful to know.

The first will use the idea that the area under a velocity versus time graph equals the change in position.



The diagram on the left shows an object transitioning from an initial velocity of v_i at t_i to a final velocity of v_f at t_f, occurring at a constant acceleration. On the right, the red area is obviously base times height or $(v_i)(\Delta t)$ and the blue area (being a triangle) is one-half base times height or $(\frac{1}{2})(\Delta t)(v_f - v_i)$. If v_f is then replaced with $v_i + a \cdot \Delta t$, the total area is change in position,

$\Delta s = (v_i)(\Delta t) + (\frac{1}{2})(\Delta t)[(v_i + a \cdot \Delta t) - v_i] = (v_i)(\Delta t) + (\frac{1}{2})(a)(\Delta t^2)$

$$\mathbf{\Delta s = (v_i)(\Delta t) + \frac{1}{2}(a)(\Delta t^2)}$$

Answer Webassign Question 3

If I then take this equation and replace Δs with $(\bar{v})(\Delta t)$ and replace a with $\frac{v_f - v_i}{\Delta t}$, it becomes

$(\bar{v})(\Delta t) = (v_i)(\Delta t) + (\frac{1}{2})(\frac{v_f - v_i}{\Delta t},)(\Delta t^2)$ which reduces to $\bar{v} = v_i + (\frac{1}{2})(v_f - v_i) = \frac{v_i + v_f}{2}$

$$\boldsymbol{\bar{v} = \frac{v_i + v_f}{2}}$$

Answer Webassign Question 4

Now this I can use if I begin with $a = \frac{v_f - v_i}{\Delta t}$, where I replace Δt with $\frac{\Delta s}{\bar{v}}$ or $\frac{\Delta s}{\frac{v_i + v_f}{2}}$

This then becomes $a = \frac{v_f^2 - v_i^2}{2 \cdot \Delta s}$ which can be rearranged to $v_f^2 = v_i^2 + 2(a)(\Delta s)$

$$\mathbf{v_f^2 = v_i^2 + 2(a)(\Delta s)}$$

Answer Webassign Question 5

One last equation is used much less frequently than the others, begins with the velocity-time graph above and $\Delta s = (v_i)(\Delta t) + (\frac{1}{2})(\Delta t)(v_f - v_i)$. Here, I'll replace v_i with $v_f - (a)(\Delta t)$ to get

$\Delta s = [v_f - (a)(\Delta t)](\Delta t) + (\frac{1}{2})(\Delta t)[v_f - (v_f - a \cdot \Delta t)] = (v_f)(\Delta t) - (a)(\Delta t^2) + \frac{1}{2}(a)(\Delta t^2) = (v_f)(\Delta t) - (\frac{1}{2})(a)(\Delta t^2)$

$$\mathbf{\Delta s = (v_f)(\Delta t) - \frac{1}{2}(a)(\Delta t^2)}$$

Disregarding the effect of air resistance, when objects move up and down near the surface of the Earth, they do so with a relatively constant acceleration. The magnitude of the acceleration is given the symbol *g* and has a value of around 9.8m/s^2. Because the acceleration is downward in most diagrams, it is calculated with -*g* or -9.8m/s^2.

If an object moves with a complexity greater than constant acceleration, this is generally left to functional notation and calculus, the steps you've seen for differentiation and integration.

As a final point, one might ask the following question: How can you say a sprinter runs to the right at 5.0m/s when different parts of their body may be moving at many different velocities?

This is valid technically, but unless otherwise specified, all objects are idealized as points in space with no internal structure. Later in the course, we will study systems which do have internal structures and which have parts which interact and move in different ways relative to a special point within the system known as the center of mass.

In review, it is good to have the following equations readily available in your memory:

For motion with constant velocity, $\mathbf{v} = \frac{\Delta s}{\Delta t}$

For motion with constant acceleration:

$$\mathbf{a} = \frac{\Delta v}{\Delta t}$$

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

$$\bar{v} = \frac{v_i + v_f}{2}$$

$$v_f^2 = v_i^2 + 2(a)(\Delta s)$$

$$\Delta s = (v_i)(\Delta t) + \frac{1}{2}(a)(\Delta t^2)$$

Very often, objects will begin with a velocity of zero and then accelerate. Under these conditions, rearrangements of the last two equations are also very useful if known immediately:

$$v_f = \pm\sqrt{2(a)(\Delta s)} \qquad \Delta s = \frac{vf^2}{(2)(a)}$$

$$\Delta s = \frac{1}{2}(a)(\Delta t^2) \qquad \Delta t = \sqrt{\frac{(2)(\Delta s)}{a}}$$