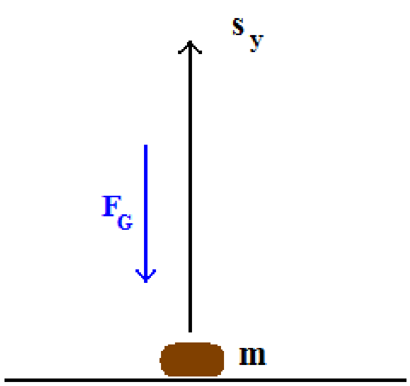
Potential energy and power

The concept of potential energy can be related to work with the idea that the work done by a conservative force equals the negative change in potential energy:

$W_{conservative} = -\Delta U$

What is a conservative force? One way of defining it is to say that the work it does is independent of the path taken by an object. This may not be very helpful, but let me just say that the two types of forces initially, spring forces and gravitational forces, are both conservative forces. So there can be spring potential energy and gravitational potential energy.

Let's look at gravitational potential energy first. Suppose you take a stone with mass *m* and lift it from the ground to a final height s_y. What is the gravitational potential energy of the stone?



First, calculate the work done by the force of gravity.

$W_G = |F_G| \cdot |\Delta s| \cdot \cos\theta = (m \cdot g)(s_y)\cos 180^o = -m \cdot g \cdot s_y$

This is equal to the negative change in potential energy, $W_{conservative} = -\Delta U$

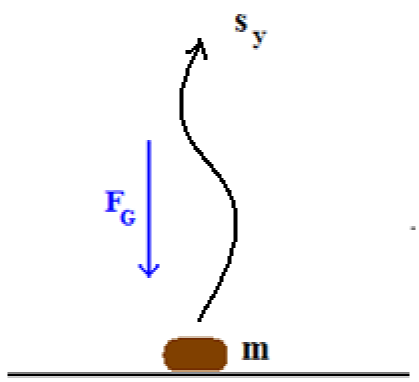
$-m \cdot g \cdot s_y = -\Delta U$ or $\Delta U = m \cdot g \cdot s_y$

s_y is more commonly known as height, *h*, so $\Delta U = m \cdot g \cdot h$

If we declare that the potential energy on the ground is zero, then the final potential energy is the same as the change and you end with an energy, $U_{gravitational} = m \cdot g \cdot h$. Technically, this potential energy is not held in the stone, but rather in the separation between the Earth and the stone.

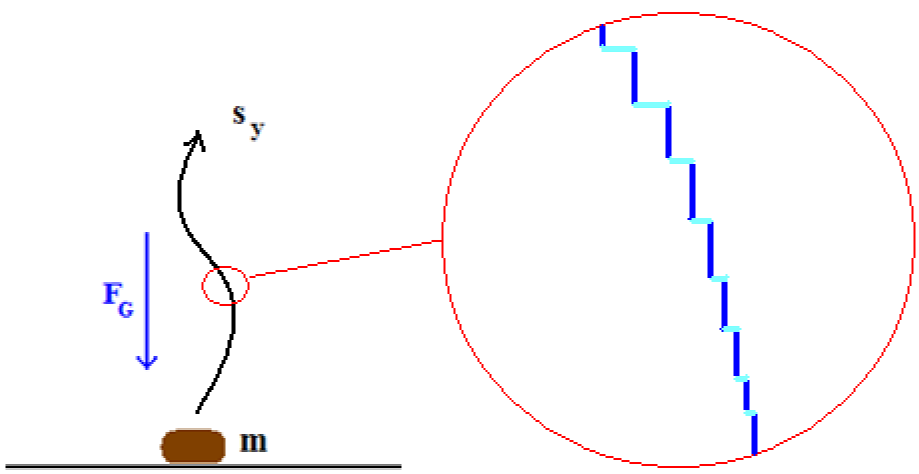
Answer Webassign Question 1

Suppose you had instead lifted the same stone to the same height along a curved path:

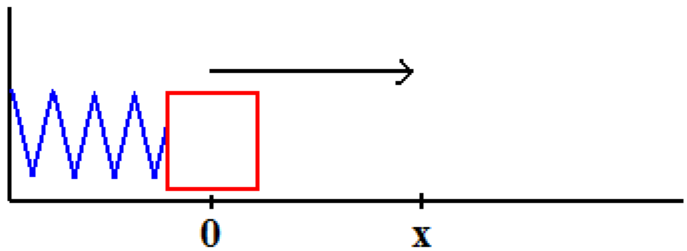


Would the work done by gravity be any different?

You can see that it won't be if you imaging zooming-in on a small section of the curve.



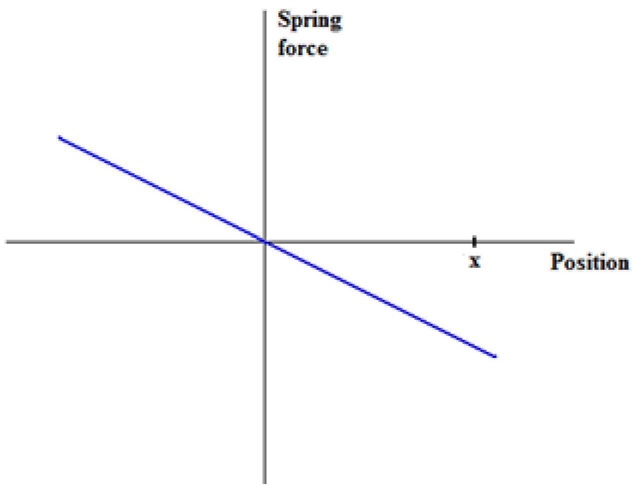
In the light-blue horizontal steps, no work is being done by gravity because the force of gravity and those little displacements are all perpendicular. But when you add up all of the dark blue vertical steps, you get the same displacement as a direct, vertical lift. So the work done is the same as if it was simply lifted straight upwards. This is the meaning of gravity being a "path independent" force.



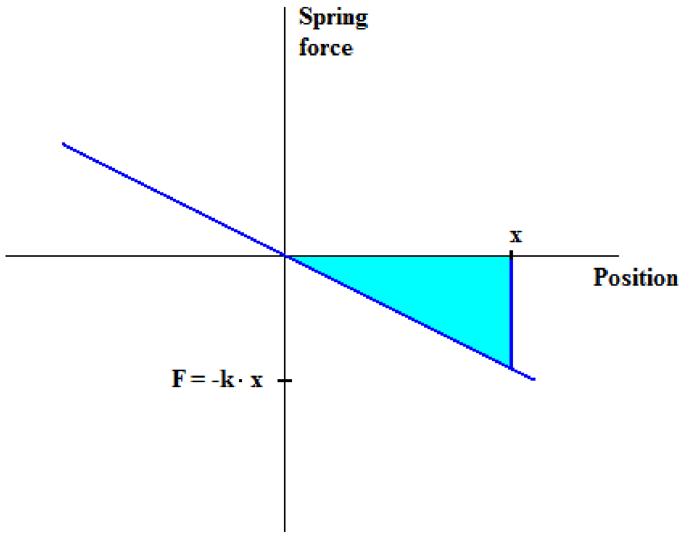
Now suppose you have a box attached to a spring, which begins unstretched at position zero. You then pull the box to a new position, s_x. How much energy will the spring store?

Again, we need to first find the work done by the spring, $W_{spring} = |F_{spring}| \cdot |\Delta s| \cdot \cos\theta$

But the spring is not going to apply a constant force; the further it is stretched, the stronger will be the force it applies. In fact, the graph of spring force versus position likely looks like this:



which follows the equation, $F_{spring} = -k \cdot x$, where k is the spring constant of the spring and $-k$ is the slope of the line. What we can do is find the area of the force-position graph and take this as the work done by the spring.



This area is $-\frac{1}{2}k\cdot x^2$ and so $W_{spring} = -\frac{1}{2}k\cdot x^2$

If $W = -\Delta U$, then $\Delta U_{spring} = \frac{1}{2}k\cdot x^2$

Again, declaring the initial potential energy of the spring to be zero,

$U_{spring} = \frac{1}{2}k\cdot x^2$

Answer Webassign Question 2

Aside from generating potential energy, work can also generate what is known as kinetic energy. For example, suppose a block with a mass of *m* sits at rest on a frictionless surface. If someone pushes the block with a force of *F* over a displacement *Δs*, they will input work to the block.

$W_{person} = F\cdot\Delta s = (m\cdot a)\cdot\Delta s$ from Newton's second law of motion

and if $v_f^2 = v_i^2 + 2(a)(\Delta s)$, then $(a)(\Delta s) = \frac{v_f^2}{2}$ when the initial velocity is zero

$W_{person} = m\cdot\frac{v_f^2}{2} = \frac{1}{2}m\cdot v_f^2$

So here the work input is producing a quantity of motion known as kinetic energy, $K = \frac{1}{2}mv^2$.

Answer Webassign Question 3

The combination of kinetic energy and potential energy is known as mechanical energy. So the net work equals the change in mechanical energy for a system.

$\Sigma W = \Delta K + \Delta U$

Lastly is the concept of power. In the same situation of someone pushing a block along a frictionless surface with a force and changing the block's kinetic energy, there was a certain amount of power input to the block by the person.

In that example, we could calculate the power input as the total work done by the person divided by the change in time in which it was input.

$P = \frac{W}{\Delta t}$

Generally, work is a way of transferring energy from one system to another *through* the application of a force. There are many other ways of transferring energy, heat, for example. So a more general definition of power is energy transferred per time and is measured in Watts, W.

$P = \frac{\Delta E}{\Delta t}$

Answer Webassign Question 4

Also, if one replaces the work in the first equation for power with $\mathbf{F} \cdot \Delta \mathbf{s}$, it becomes

$P = \frac{\mathbf{F} \cdot \Delta \mathbf{s}}{\Delta t} = \mathbf{F} \cdot \mathbf{v}$

Power at some instant in time is the force applied to an object times the velocity of that object.

Answer Webassign Question 5

To review:

$U_{gravitational} = m \cdot g \cdot h$ $\quad$ $U_{spring} = \frac{1}{2} k \cdot x^2$ $\quad$ $K = \frac{1}{2} m v^2$

$P = \frac{\Delta E}{\Delta t}$ $\quad$ or $\quad$ $P = \frac{W}{\Delta t} = \mathbf{F} \cdot \mathbf{v}$