

Projectiles

A projectile is basically an object that flies through the air under the lone influence of the force of gravity. It's not propelled like a rocket and, unless specified otherwise, air resistance is taken to be negligible. A small stone thrown in a game of hop-scotch is a simple projectile.

The key idea is that, at least in classical physics, the horizontal motion in the x-dimension is independent of the vertical motion in the y-dimension. So the horizontal motion is motion with constant velocity and the vertical motion is motion with constant acceleration, namely $-g$ or -9.8m/s^2 .

As with vectors in two dimensions, you want to take any initial information and decompose it into x and y components. You can then apply kinematics equations to each dimension independently to find further information about the motion in that dimension. At the end, you may be required to recompose that information.

Let me clarify all of that with an example: An archer stands on a tower and launches an arrow from a height of 5.0m above the ground. The arrow leaves the bow with a velocity of (100m/s, 20°). How far from the tower will it land and with what velocity will it land?

Step 1: Take all information that needs to be divided into x and y components and do so. Here that is only the initial velocity of (100m/s, 20°).

$$\text{Initial } v_x = (100\text{m/s})(\cos 20^\circ) = 94\text{m/s}$$

$$\text{Initial } v_y = (100\text{m/s})(\sin 20^\circ) = 34\text{m/s}$$

Step 2: Fill-in known values into kinematics boxes. Because the x-dimension has motion with constant velocity, there are only three quantities to consider. The y-dimension has motion with constant acceleration, so it has six.

x dimension	y dimension
$\Delta s =$	$\Delta s = -5\text{m}$ $\bar{v} =$
$v = 94\text{m/s}$	$v_i = 34\text{m/s}$ $a = -9.8\text{m/s}^2$
$\Delta t =$	$v_f =$ $\Delta t =$

Even though the arrow will rise and then fall, the overall vertical change in position is from +5m to 0m, which is a displacement of -5m.

Step 3: Use kinematics equations to calculate the unknowns. Here, the v_f^2 equation will provide the final velocity (negative because the arrow is descending when it lands), then the average velocity and time are fairly easy to find.

x dimension	y dimension
$\Delta s =$	$\Delta s = -5\text{m}$ $\bar{v} = -0.7\text{m/s}$
$v = 94\text{m/s}$	$v_i = 34\text{m/s}$ $a = -9.8\text{m/s}^2$
$\Delta t =$	$v_f = -35.4\text{m/s}$ $\Delta t = 7.08\text{s}$

Once the time of flight has been calculated in one dimension, it must be the same in the other because the arrow has one single flight. That makes horizontal displacement easy to find.

x dimension	y dimension
$\Delta s = 665.5\text{m}$	$\Delta s = -5\text{m}$ $\bar{v} = -0.7\text{m/s}$
$v = 94\text{m/s}$	$v_i = 34\text{m/s}$ $a = -9.8\text{m/s}^2$
$\Delta t = 7.08\text{s}$	$v_f = -35.4\text{m/s}$ $\Delta t = 7.08\text{s}$

Step 4: Answer whatever questions have been asked. Here, the arrow will land about 665.5m from the base of the tower. The overall velocity it lands with comes from unifying 94m/s and -35.4m/s with the Pythagorean Theorem and this is about 100.4m/s.

The corresponding angle of this velocity comes from $\tan^{-1} \left(\frac{v_y}{v_x} \right)$ or $\tan^{-1} \left(\frac{-35.4}{94} \right)$ and this is about 339.4° (your calculator output plus the 360° correction factor).

Answer Webassign Question 1

Answer Webassign Question 2

Answer Webassign Question 3

Answer Webassign Question 4

Answer Webassign Question 5