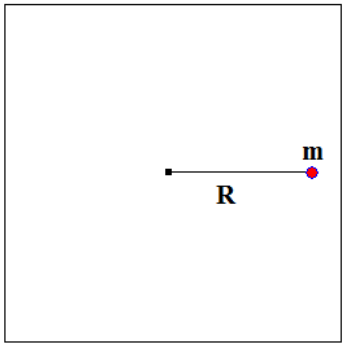
Rotational inertia

Suppose we have a simple system with a puck of mass *m* tied to a string of length *R* sitting on a frictionless table:



view from above

Now let's say you push on the puck with a force of *F* (in a direction of 90º relative to the diagram).

$F = m \cdot a$

and from a previous derivation, $a = \alpha \cdot R$, so

$F = m \cdot \alpha \cdot R$

We can also multiply both sides by R to get

$R \cdot F = [m \cdot R^2] \cdot \alpha$ or $\tau = [m \cdot R^2] \cdot \alpha$

Clearly, the angular acceleration is proportional to the torque applied and inversely proportional to this bracket of $[m \cdot R^2]$.

The bracket therefore represents the resistance to angular acceleration and is called the rotational inertia, I.

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The same way masses can be added together, rotational inertias can be added together, so

$I_{system} = m_1R_1^2 + m_2R_2^2 + m_3R_3^2 + \ldots\ldots = \sum m \cdot R^2$

This is true for point masses or masses which are small relative to the radius in which they move.

If the mass is distributed through space, like a disk that spins around itself, then calculus notation must be used and you will simply be given the equation for that particular geometry. For instance, a solid disk has a rotational inertia of $I_{disk} = \frac{1}{2}MR^2$ where M is the total mass of the disk and R is the radius of the disk. For a bar that rotates around its end, $I_{bar} = \frac{1}{3}ML^2$ where M is the mass of the bar and L is the length.

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Also, torque is a vector and vectors can be added together, so we can write $\tau = [m \cdot R^2] \cdot \alpha$ as

$\boldsymbol{\tau} = I \cdot \boldsymbol{\alpha}$ where $\boldsymbol{\tau}$ implies net torque

This is the rotational analog of $\mathbf{F} = m \cdot \mathbf{a}$

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Our chart relating linear motion and rotational motion can then be continued:

Linear kinematics concept	Rotational kinematics concept
Linear position – s	Angular position – θ
Linear velocity – v	Angular velocity – ω
Linear acceleration – a	Angular acceleration – α
Force: F	Torque: $\boldsymbol{\tau} = \mathbf{R} \times \mathbf{F}$
Mass: m	Rotational inertia: $I = \sum m \cdot R^2$
$\mathbf{F} = m \cdot \mathbf{a}$	$\boldsymbol{\tau} = I \cdot \boldsymbol{\alpha}$

Additionally, if $\Sigma K = \frac{1}{2} m \cdot v^2$ and, for rotating particles, $v = \omega \cdot R$, then

$\Sigma K = \frac{1}{2} m \cdot v^2 = \frac{1}{2} m(\omega \cdot R)^2 = \frac{1}{2}[\Sigma\, m \cdot R^2] \cdot \omega^2 = \frac{1}{2} I \cdot \omega^2$

And in the same way we defined linear work as $\mathbf{F} \cdot \Delta \mathbf{s}$, we can define rotational work as $\boldsymbol{\tau} \cdot \Delta \boldsymbol{\theta}$.

An input of rotational work to a system can cause a change in its rotational kinetic energy by:

$\boldsymbol{\tau} \cdot \Delta \boldsymbol{\theta} = (I \cdot \boldsymbol{\alpha}) \cdot \frac{\boldsymbol{\omega_f^2 - \omega_i^2}}{\boldsymbol{2 \cdot \alpha}} = \frac{1}{2} I \cdot \omega_f^2 - \frac{1}{2} I \cdot \omega_i^2 = \Delta K_{rotational}$

Therefore,

Linear kinematics concept	Rotational kinematics concept
Linear kinetic energy: $K = \frac{1}{2} m \cdot v^2$	Rotational kinetic energy: $K = \frac{1}{2} I \cdot \omega^2$
Linear work: $W = \mathbf{F} \cdot \Delta \mathbf{s}$	Rotational work: $W = \boldsymbol{\tau} \cdot \Delta \boldsymbol{\theta}$

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