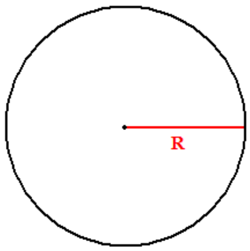
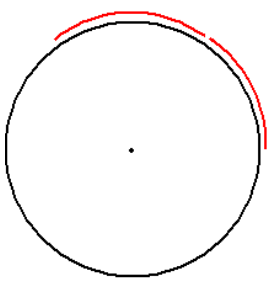
Rotational kinematics

Suppose you cut a circle out of a piece of paper and then several pieces of string which are just as long as the radius of the paper circle.



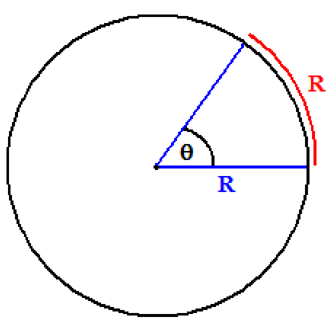
If you then begin to lay these pieces of string end-to-end around the circumference of the circle, how many pieces of string will it take to go all the way around?



The answer is that it would take about 6.28 pieces of string or, more precisely, 2π pieces.

This is because, regardless of the size of the circle, the distance of the circumference of a circle is 2π times the distance of the radius.

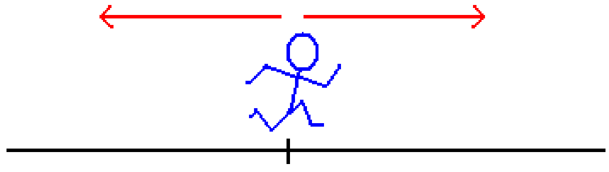
We can then define one radian as the angle that sits underneath (or subtends) one of these pieces of string.

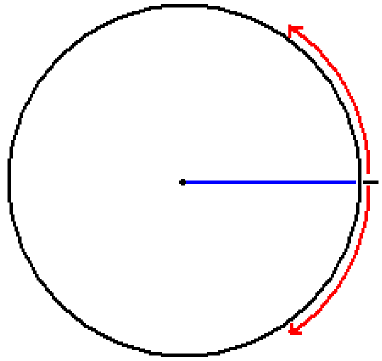


Clearly, if it takes 2π pieces of string to go around an entire circle and an angle of one radian sits under each piece of string, then there is an angular measurement of 2π radians in one full circle.

Altogether, one full rotation equals 360º equals 2π radians.

Answer Webassign Question 1





In the same way we described linear motion with the concepts of linear position, velocity, and acceleration, we can describe rotational motion with the concepts of angular position, velocity, and acceleration. This can easily be done is a small chart.

Linear kinematics concept	Rotational kinematics concept
Linear position: s – how far the runner is left (-) or right (+) of the origin, units of meters	Angular position: θ – the position of the disk clockwise (-) or counterclockwise (+) from the positive x-axis, units of radians
Linear velocity: v – change in linear position divided by corresponding change in time, $v = \frac{\Delta s}{\Delta t}$, units of m/s	Angular velocity: ω - change in angular position divided by corresponding change in time, $\omega = \frac{\Delta\theta}{\Delta t}$, units of rad/s
Linear acceleration: a – change in linear velocity divided by corresponding change in time $a = \frac{\Delta v}{\Delta t}$, units of m/s^2	Angular acceleration: α – change in angular velocity divided by corresponding change in time $\alpha = \frac{\Delta\omega}{\Delta t}$, units of rad/s^2
If the linear velocity is not constant, $\bar{v} = \frac{\Delta s}{\Delta t}$	If the angular velocity is not constant, $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$
If linear acceleration is constant:	If angular acceleration is constant:
$\bar{v} = \frac{v_i + v_f}{2}$	$\bar{\omega} = \frac{\omega_i + \omega_f}{2}$
$v_f^2 = v_i^2 + 2(a)(\Delta s)$	$\omega_f^2 = \omega_i^2 + 2(\alpha)(\Delta\theta)$
$\Delta s = (v_i)(\Delta t) + \frac{1}{2}(a)(\Delta t^2)$	$\Delta\theta = (\omega_i)(\Delta t) + \frac{1}{2}(\alpha)(\Delta t^2)$

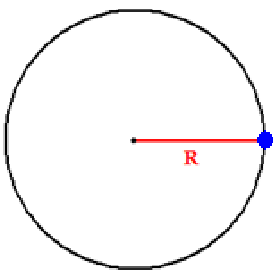
Answer Webassign Question 2

When something rotates counter-clockwise, there is a vector associated with this rotation determined by the right hand rule for rotation. Curl the fingers of your right hand counter-clockwise and you will that your thumb points out of the screen, aligning the positive z-axis with positive rotation. If you curl the fingers of your right hand clockwise, your thumb will point into the screen along the negative z-axis.

Later, we will see all of the dynamics concepts we've had for linear motion will also have rotational analogs.

Inertial mass	Rotational inertia
Force	Torque
Linear work	Rotational work
Linear kinetic energy	Rotational kinetic energy
Linear impulse	Angular impulse
Linear momentum	Angular momentum

Now suppose we have a disk with a radius *R* that is free to spin and we put a small blue sticker on the edge.



If we rotate the disk *x* number of radians, what distance will the sticker travel? Well, we know by the definition of a radian, for every radian rotated, the sticker is going to travel an arc length equal to one radius of distance.

So, arc length = $\Delta\theta \cdot R$

If this happens in certain amount of time, Δt, we can divide both sides by Δt and get

$\frac{arc\ length}{\Delta t} = \frac{\Delta\theta}{\Delta t} \cdot R$ or $v = \omega \cdot R$

Answer Webassign Question 3

Now suppose the disk speeds-up or slows-down. Both v and ω will change over a time span, Δt.

$\frac{\Delta v}{\Delta t} = \frac{\Delta\omega}{\Delta t} \cdot R$ or $a = \alpha \cdot R$

This a is sometimes written as a_T to specify it is tangential acceleration, as opposed to centripetal acceleration.

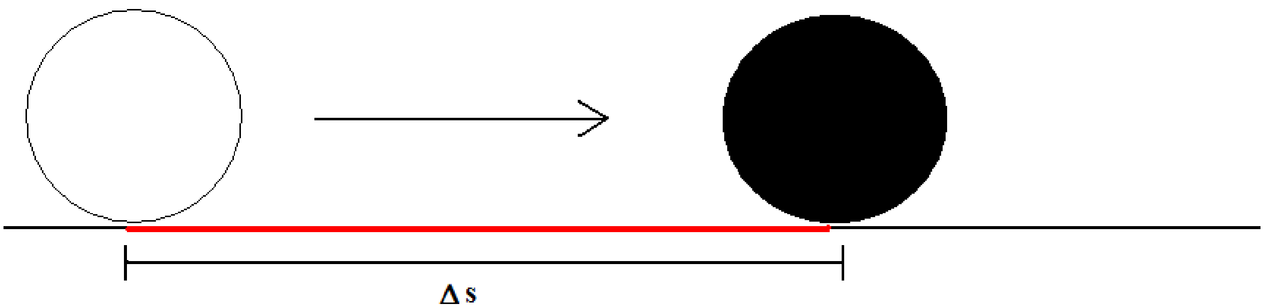
Incidentally, for centripetal acceleration, if $a_c = \frac{v^2}{R}$ and $v = \omega \cdot R$

Then $a_c = \frac{(\omega \cdot R)^2}{R} = \omega^2 \cdot R$

Answer Webassign Question 4

Now suppose you take a metal cylinder and cover it in red paint.

If you roll the cylinder to the right, how far, Δs, will it travel before it rolls off one layer of paint?



Clearly, the length of the red line is one circumference of distance, or $2\pi \cdot R$. The cylinder itself make one complete rotation, so its angular displacement was 2π radians.

Therefore, for this situation of rolling without slipping, we can write $\Delta s = \Delta\theta \cdot R$.

We can subscript the *s* with a little T for translational motion or with a little CM to indicate it is the center of mass of the cylinder that has moved this displacement.

Answer Webassign Question 5

Then, just like before, we can divide each side by Δt to get an equation for velocity, then again for an equation for acceleration. Altogether, it will provide three equations for rolling without slipping:

$\Delta s_{cm} = \Delta\theta \cdot R$

$v_{cm} = \omega \cdot R$

$a_{cm} = \alpha \cdot R$

If the cylinder does slip, then these equations do not hold true. For example, if you are stuck in snow and spinning-out your tires, you have a very large ω for the tires, but zero v_{cm}.

Or if you are driving on ice, your brakes lock, and you skid along the ice, you have zero ω but a non-zero v_{cm}.

To review:

One radian is the angle underneath an arc length equal to the radius of the circle, meaning there are 2π radians in every one rotation.

All linear kinematics concepts have rotational analogs. Position, velocity, and acceleration are s, v, and a for linear motion, θ, ω, and α for angular motion.

For rolling without slipping, three equations specify the relationship between the object's center of mass moving left and right and the object spinning around itself:

$\Delta s_{cm} = \Delta\theta \cdot R$

$v_{cm} = \omega \cdot R$

$a_{cm} = \alpha \cdot R$