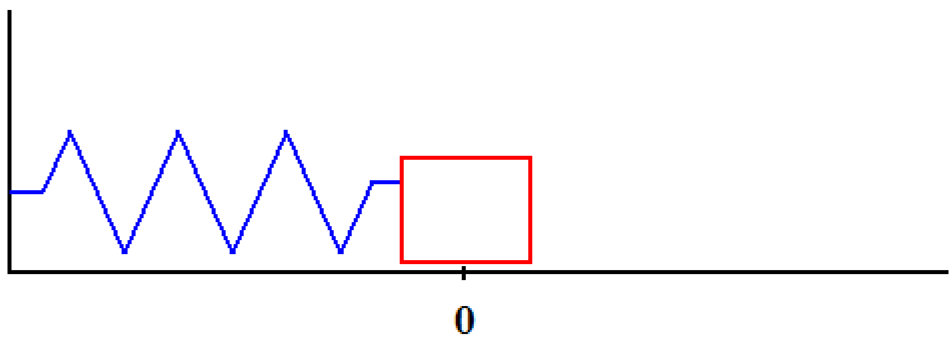
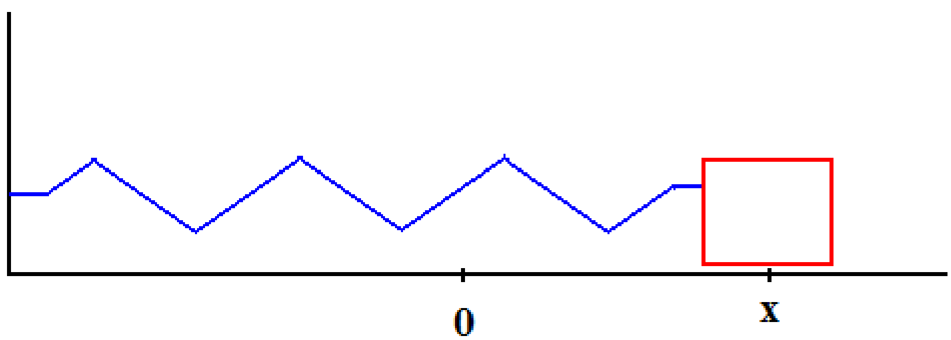
Springs

Suppose you take a spring and, oriented horizontally, you attach the left side of the spring to fixed wall and the right side to a small block.

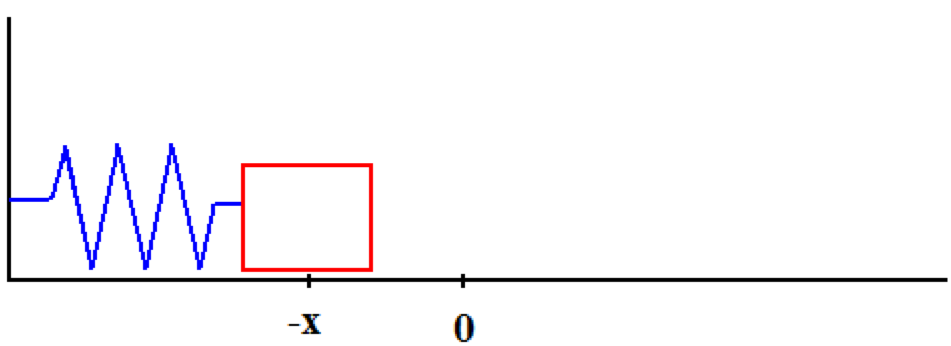


If the spring is neither compressed nor stretched, we will define the position of the block to be zero. We can then pull the block to the right to a position x and use a force-meter to determine the force that the spring applies when stretched this amount.



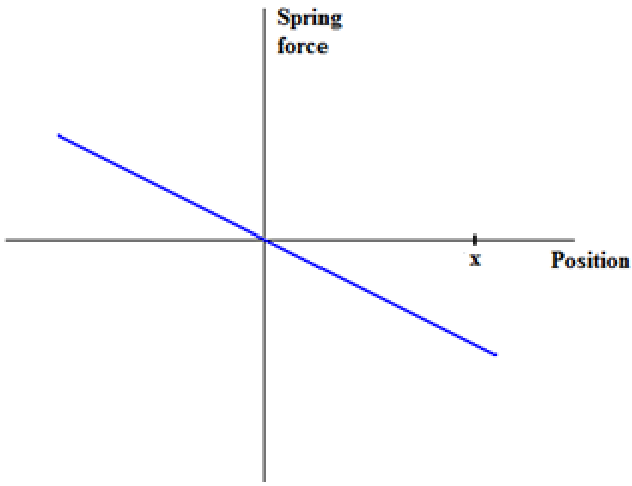
For many springs, the force the spring pulls back to the left with increase as *x* increases. The more the spring is stretched, the harder it pulls back leftward with a negative force.

Likewise, if you push the box to the left, the further it is pushed left, the harder it will apply a force on the box to the right.



If this relationship between force and position is linear, the spring is said to follow Hooke's law, which you should assume to be true of springs unless something else is specified.

So the linear relationship would make a graph like this:



Again, you can see that when the box has a negative position, the spring applies a positive force (in the second quadrant of the graph) and when the box has a positive position, the spring applies a negative force (in the fourth quadrant of the graph).

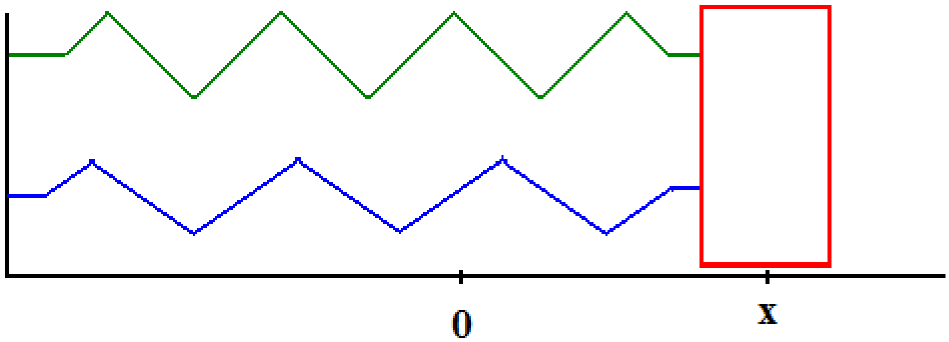
The equation of this graph is then written as $F_{spring} = -k \cdot x$, where $-k$ is the slope of the line and k is known as the spring constant of the spring. x is the amount the spring has been stretched or compressed, sometimes written as $(L - L_0)$, length minus original length.

Answer Webassign Question 1

Answer Webassign Question 2

Answer Webassign Question 3

Now suppose you had two springs, one with a spring constant k_1 and one with a spring constant k_2. If both springs are attached to a block parallel to each other and the block is pulled right, what forces will they apply?



The top spring will apply a force of $-k_1 \cdot x$ on the block, the bottom spring will apply a force of $-k_2 \cdot x$ on the block, making the net force on the block $(-k_1 \cdot x) + (-k_2 \cdot x)$.

Let's imagine these two springs are acting as a single spring with a single spring constant, called the "effective spring constant" of the system. What is this effective spring constant? Well, go back to Hooke's law:

$F_{spring\ system} = -k_{effective} \cdot x$

We just determined that the spring system will apply a force of $(-k_1 \cdot x) + (-k_2 \cdot x)$. Therefore,

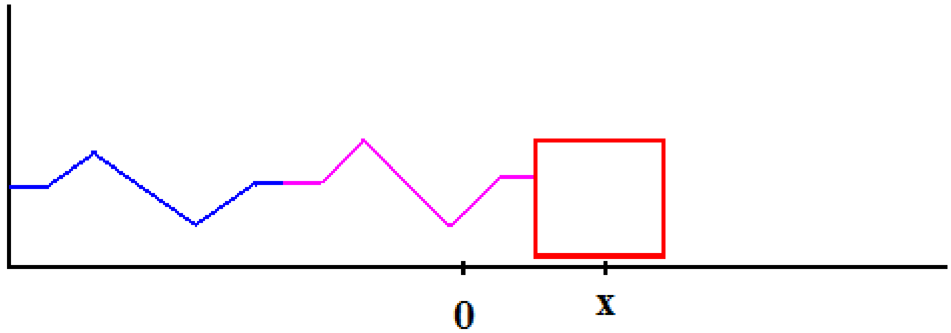
$(-k_1 \cdot x) + (-k_2 \cdot x) = -k_{effective} \cdot x$

$k_{effective} = k_1 + k_2$

If we had more springs in parallel, it would just continue: $k_{effective} = k_1 + k_2 + k_3 + \ldots..$

Answer Webassign Question 4

Suppose we instead construct the system so that, from left to right, it is wall - spring one - spring two – block.



We know that *x* in this diagram is the sum of the stretching of the two springs, which we can call $x_1 + x_2$. We can then rearrange Hooke's law into $x = \frac{-F}{k}$.

$x = x_1 + x_2$

$$\frac{-F_{system}}{k_{effective}} = \frac{-F_1}{k_1} + \frac{-F_2}{k_2}$$

F_{system} is the force the system is applying to the box, which is clearly the same as F_2, the force the right spring is applying to the box because the right spring is the only spring applying a force to the box.

Also, F_1 is the same as F_2, otherwise the point in the system connecting the two springs would be accelerating. So if $F_{system} = F_1 = F_2$, the equation simply becomes:

$$\frac{1}{k_{effective}} = \frac{1}{k_1} + \frac{1}{k_2}$$

And again, if there were more than two springs, it would just continue:

$$\frac{1}{k_{effective}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots\ldots..$$

This is the equation for what is called "springs in series". One spring is after another, after another, etc. Like the World Series, the games are sequential, not all played at the same time.

Answer Webassign Question 5

To review:

$F_{spring} = -k \cdot x$

k is the spring constant and x is the amount stretched or compressed

Springs in parallel: $k_{effective} = k_1 + k_2 + k_3 + \ldots..$

Springs in series: $\frac{1}{k_{effective}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots\ldots..$