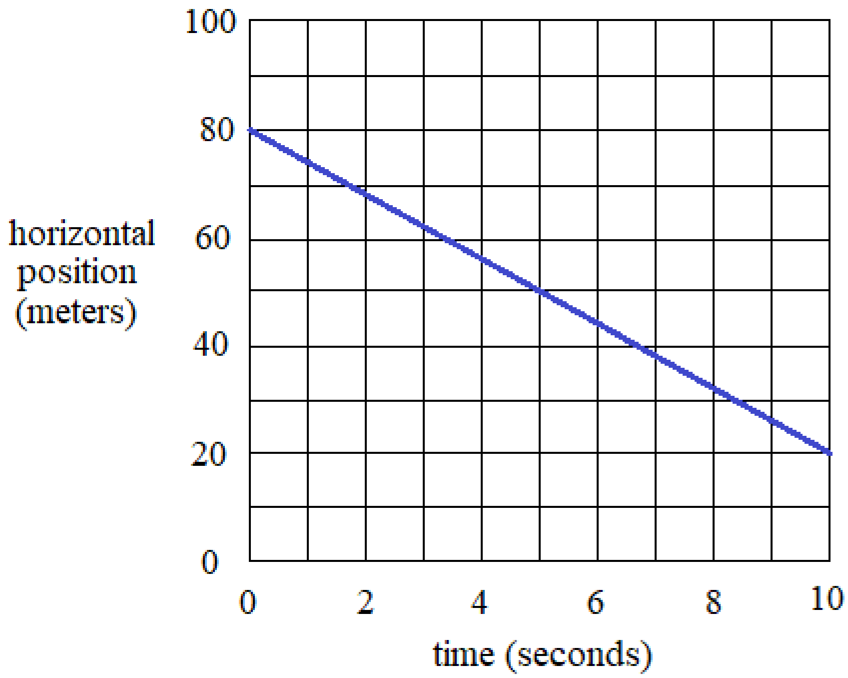
AP Physics 1 – Summer Tutorial 2

Translating between graphs



To review the ideas of the first tutorial, see if you can answer the following questions. The answers will be on the next pages.

1. What is the velocity from 5s to 10s, including units?

2. What is the velocity at the instant of 2s?

3. What is the speed from 0s to 5s?

4. What is the speed at the instant of 8s?

5. What is equation of the line?

6. What is the position at 8s?

7. At what time is the position 40.4m?

8. What is the displacement between 0s and 5s?

9. Is the motion leftward or rightward?

10. If you made a corresponding graph of velocity versus time, what would that graph look like?

Answers:

1. The velocity is the slope of the line from 5s to 10s. At 5s, the position is 50m and at 10s, the position is 20m. To go from 50m to 20m is a change of -30m along the y-axis. To go from 5s to 10s is a change of 5s along the x-axis.

Slope = $\frac{\Delta y}{\Delta x}$ = $\frac{-30m}{5s}$ = -6m/s

2. The slope is constant throughout the motion, so the velocity at any instant in time (usually called the *instantaneous velocity*) is also -6m/s.

3. Speed is the absolute value of velocity, so the speed is 6m/s.

4. The *instantaneous speed* is 6m/s at any time in the graph shown.

5. In the form of $y = mx + b$, we would have $s_x = -6 \cdot t + 80$.

The slope is -6m/s, the y-intercept is 80m. "s" is the variable for position and the "x" subscript specifies that the positions are horizontal.

6. Using $s_x = -6 \cdot t + 80$ where t = 8s produces a position of 32 meters.

7. Using $s_x = -6 \cdot t + 80$ where s_x = 40.4m produces a time of 6.6 seconds.

8. Displacement means change in position. To go from 80m to 50m is a change of -30m.

In general, the change of some value is the final value minus the initial value. So here:

$\Delta s = s_{final} - s_{initial}$ = 50m – 80m = -30m

You could also have used the known velocity and the known time span.

Defining velocity as the slope of a position versus time graph produces the equation:

velocity = $\frac{change\ in\ position}{change\ in\ time}$ or $v = \frac{\Delta s}{\Delta t}$

Given a velocity of -6m/s and a time span of 5s, we can solve for the displacement.

-6m/s = $\frac{\Delta s}{5s}$ so Δs = -30m

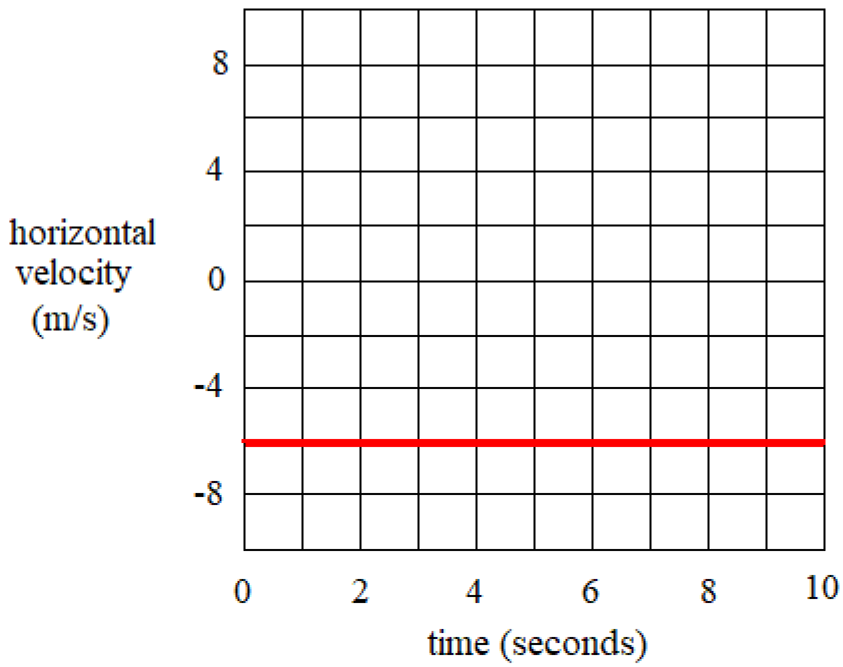
Remember this equation and its common rearrangement:

velocity = $\frac{change\ in\ position}{change\ in\ time}$ or $v = \frac{\Delta s}{\Delta t}$

change in position = (velocity)·(change in time) or $\Delta s = v \cdot \Delta t$

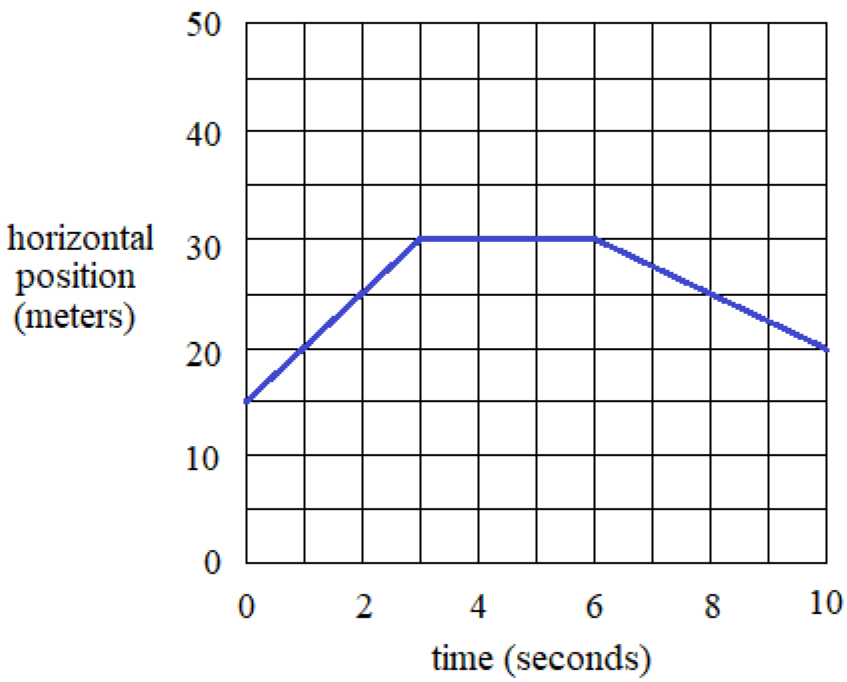
9. Using the standard convention that, for horizontal motion, rightward is positive and leftward is negative, a negative slope implies leftward motion.

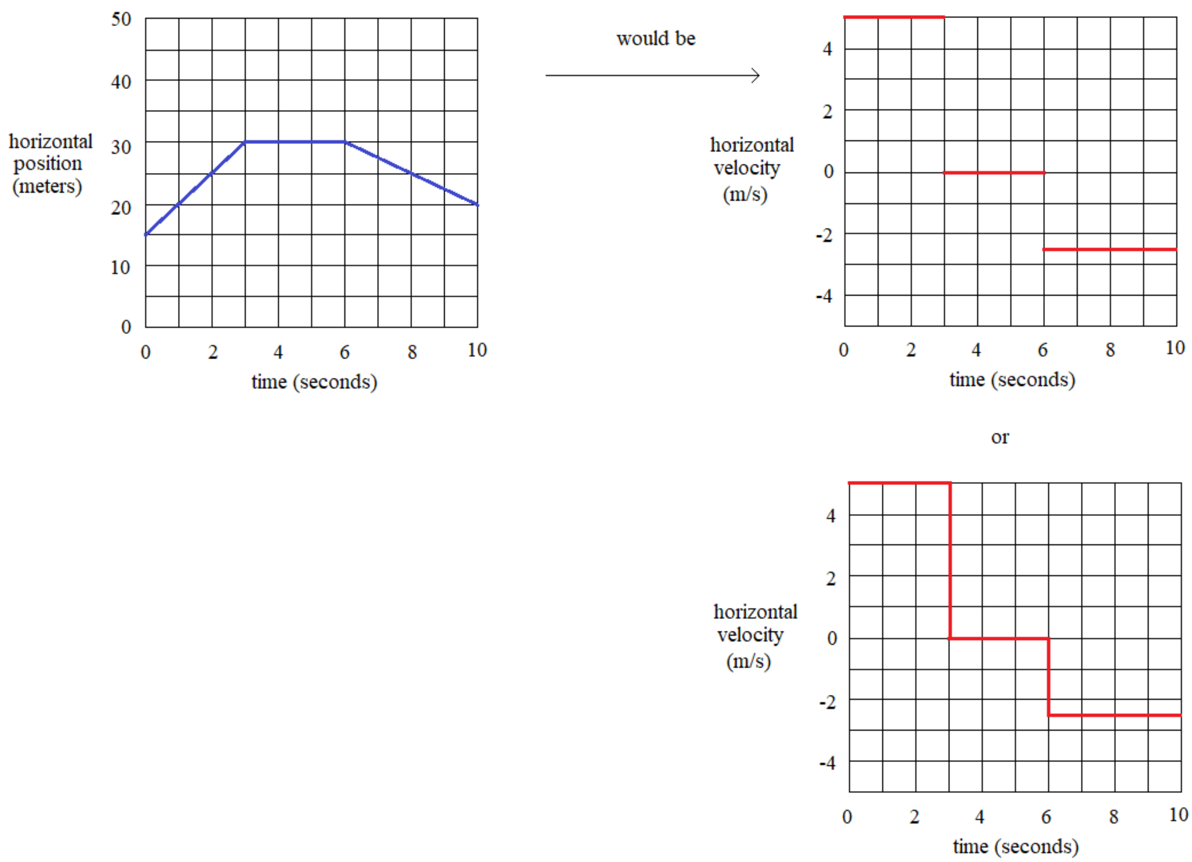
10. The provided graph shows a velocity of -6m/s that occurs from 0s to 10s, so the velocity graph would be:



This is just a horizontal line at a value of -6m/s. So, as a rule, the slope of a position versus time graph becomes the value of the velocity versus time graph.

For practice, try translating the position versus time graph below into a velocity versus time graph. The answer is on the next page.

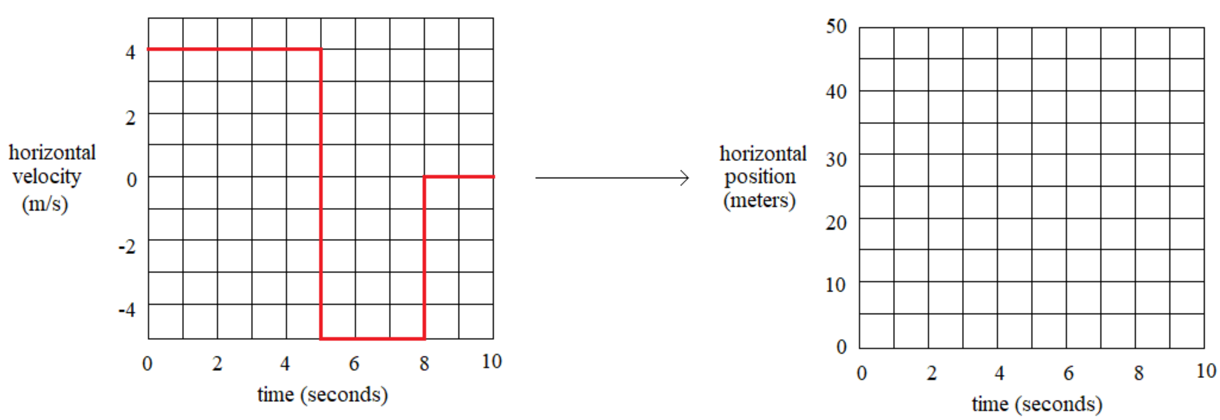




It doesn't matter significantly if you do or do not include the vertical connecting lines.

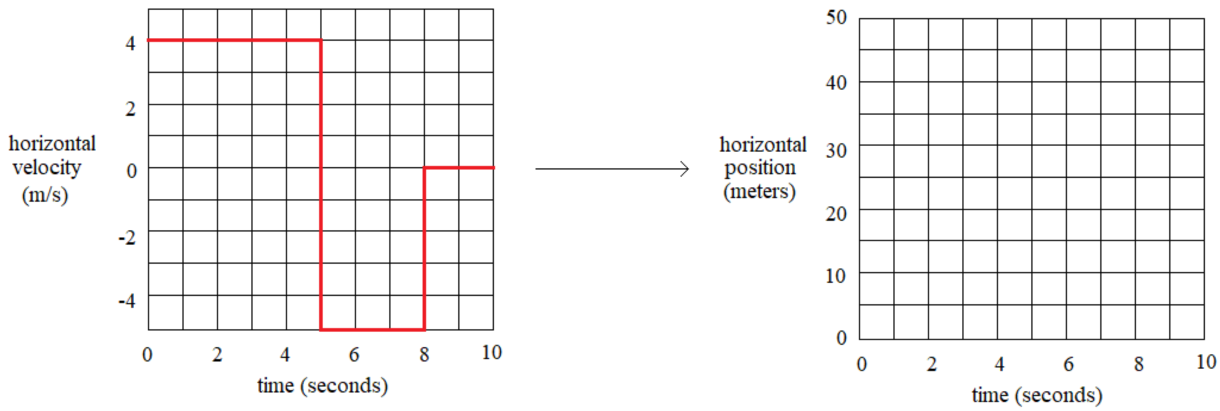
A short way or remembering these translations is to think "the slopes of the position graph become the steps of the velocity graph." For example, in first three seconds, we had a slope of 5m/s on the position graph, therefore there is a step at a value of 5m/s on the velocity graph.

How about translating backwards, from a known velocity graph to a position graph? You must be given the initial position and here we'll say **the initial position is 10m**.



You may want to try the above translation yourself and see if you can figure-it-out before going on to the next page.

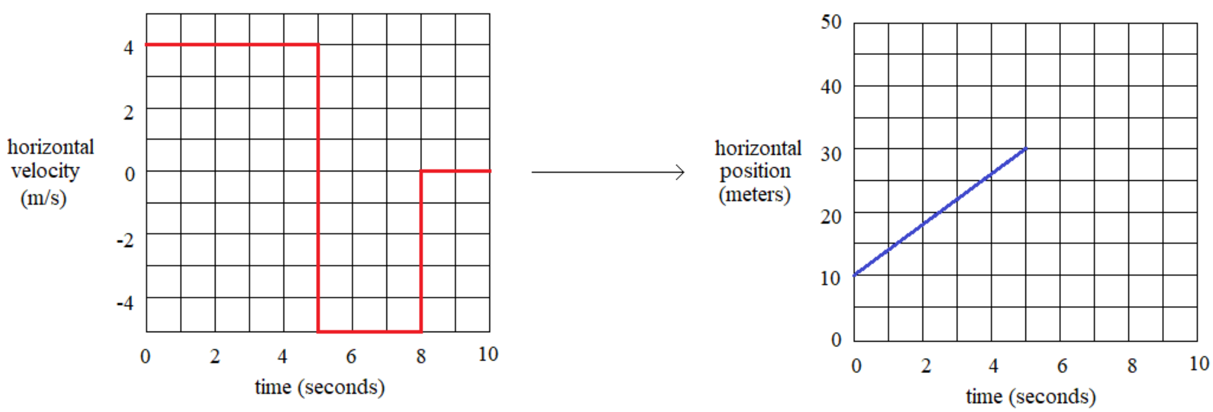
There are a few ways to do this.



One way is to take the first line segment from 0s to 5s and notice the velocity is 4m/s. What does this 4m/s mean? It's a ratio of change in position per change in time:

the position changes by +4m for every second of time that passes

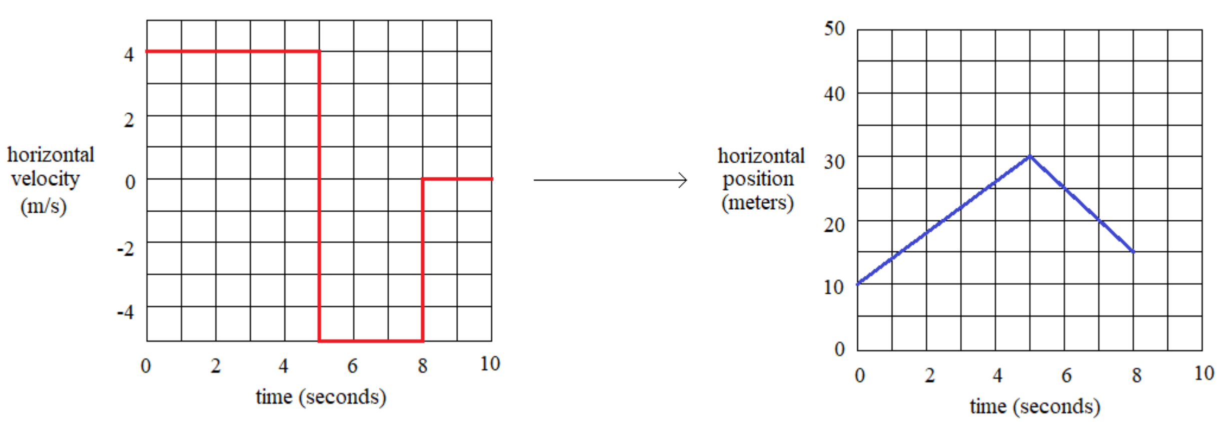
If five seconds of time pass, then the total change in position would be +20m, and that's what you graph over those first five seconds (with the reminder the initial position was given as 10m):



Go on to the next line segment of the velocity graph, from 5s to 8s. Here the velocity is -5m/s:

the position changes by -5m for every second of time that passes

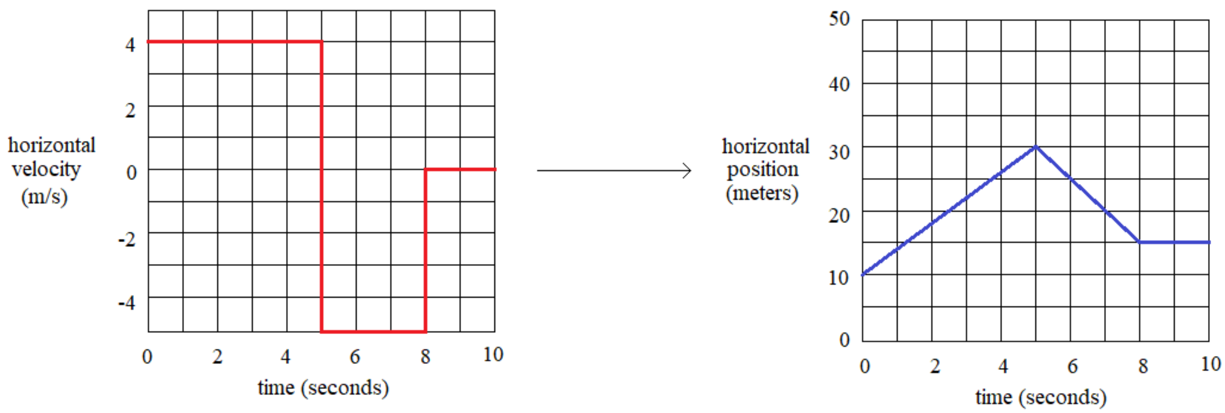
If three seconds of time pass, then this change in position is -15m from where the graph left-off during that same time span, from 5s to 8s:



And lastly, from 8s to 10s on the velocity graph, we have a slope of 0m/s:

the position changes by 0m for every second of time that passes

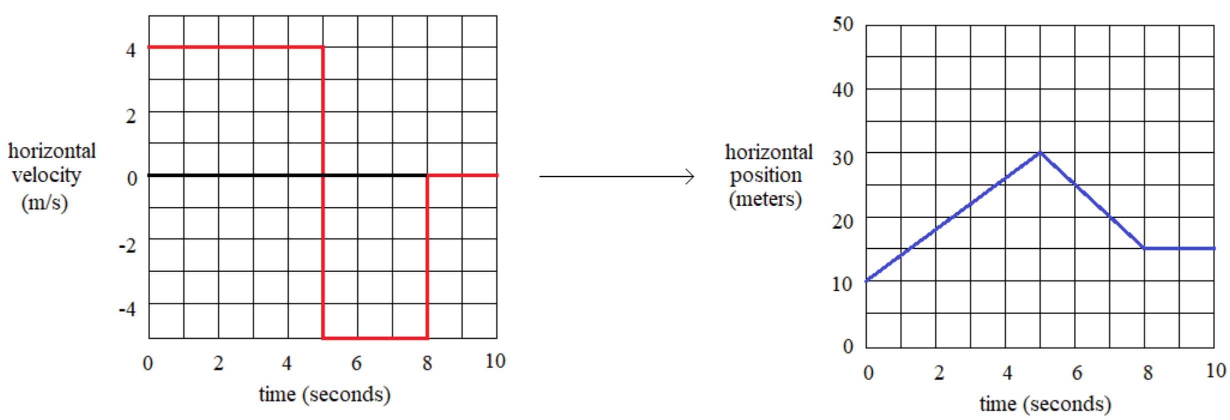
If two seconds pass, the change in position is 0m from where the graph left-off during that time span from 8s to 10s:



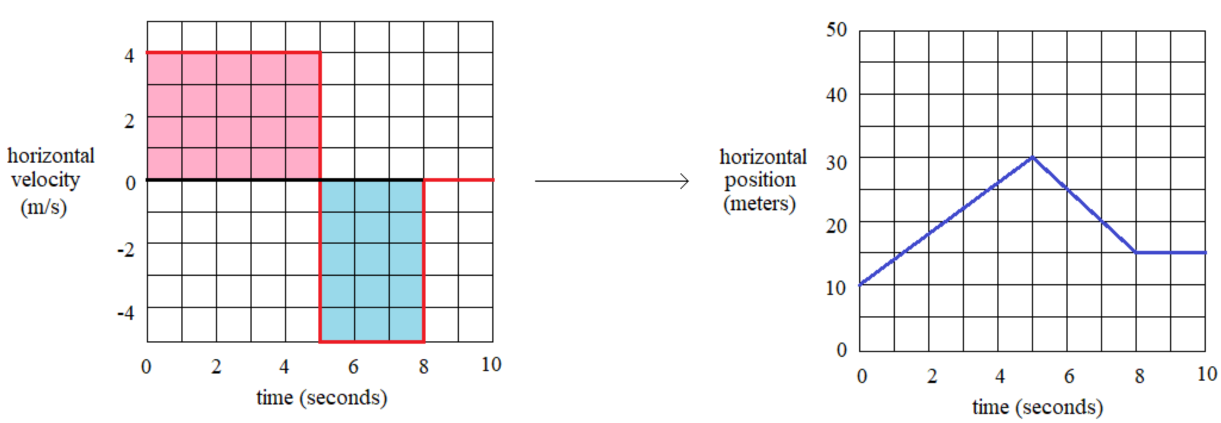
Another way to do this, which turns-out to be very useful in more complex situations is the following principle:

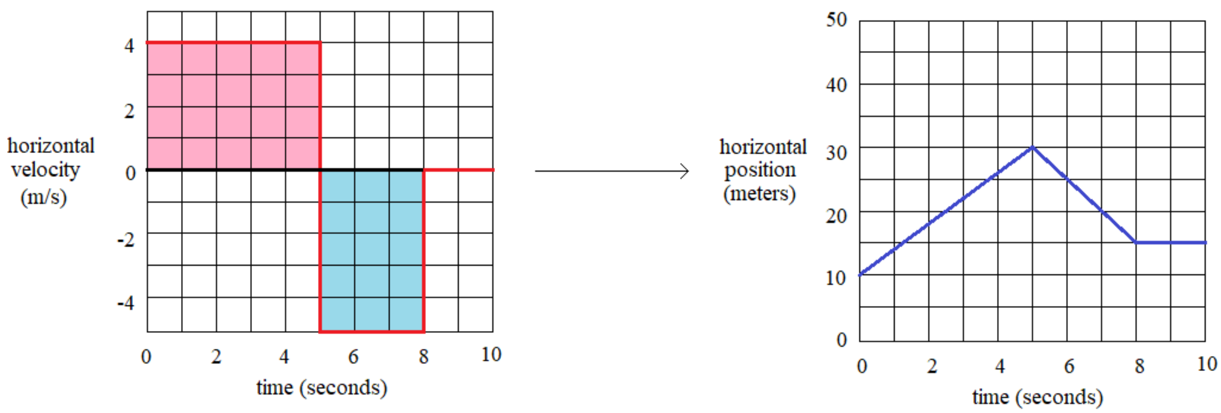
The area trapped by a velocity function equals the change in position.

By “area trapped”, we mean the area between the line of the function and the x-axis, so it can be helpful to darken-in the x-axis on the velocity graph as a reference:



Now I’ll shade-in the areas trapped for the given time spans.





Looking at the velocity graph from 0s to 5s, we have shaded a rectangle which has a base of 5s and a height of 4m/s.

Area = (base)·(height) = (5s)·(4m/s) = 20m

which is exactly the change in position on the position graph during that same time, from 0s to 5s

Likewise, from 5s to 8s:

Area = (base)·(height) = (3s)·(-5m/s) = -15m also the displacement seen from 5s to 8s

Even for 8s to 10s, when there is no area trapped because the function sits on the x-axis, this agrees with the zero displacement from 8s to 10s on the position graph.

So now we have two very useful principles of translation:

The slope of a position versus time graph is the velocity

The area trapped in a velocity versus time graph is the change in position