AP Physics 1 – Summer Tutorial 3

Proportional reasoning

Let's use the equation $\Delta s = v \cdot \Delta t$

read as: *change in position equals velocity multiplied by change in time*

We might ask the question, "What happens to the change in position if you double the velocity, keeping the time span constant?"

It may be already obvious that the change in position doubles, but let's formulate a process that will work when such questions become more complex.

1. Start with the equation: $\Delta s = v \cdot \Delta t$

2. Replace any variable with the given change in the variable. Here, we are told that the velocity doubles, so we replace the *v* with the number two: $\Delta s = [2] \cdot \Delta t$

3. Replace any constant variables with a one. We are told the time span remains constant, so we replace the *Δt* with the number one: $\Delta s = [2] \cdot [1]$

4. Solve for the quantity in question. Here, this is $\Delta s = [2] \cdot [1] = 2$

$\Delta s = 2$ or "change in position doubles"

Likewise, if the velocity tripled and the time quadrupled, we would have:

$\Delta s = v \cdot \Delta t$

$\Delta s = [3] \cdot [4]$

$\Delta s = 12$ or "change in position increases by a factor of twelve"

The process works just as well with reductions. If the velocity is cut in half and the time is reduced to one-sixteenth the original time:

$\Delta s = v \cdot \Delta t$

$$\Delta s = \left[\frac{1}{2}\right] \cdot \left[\frac{1}{16}\right]$$

$\Delta s = \frac{1}{32}$ or "change in position decreases by a factor of 32".

What must be the change in velocity if the time triples and the change in position diminishes by a factor of four?

$\Delta s = v \cdot \Delta t$

$\left[\frac{1}{4}\right] = v \cdot [3]$

$v = \left[\frac{1}{12}\right]$ or the velocity diminishes by a factor of twelve

Here are a few more examples:

1. The area of a triangle follows the equation: $A = \frac{1}{2}(base)(height)$

If the base doubles and the height triples, what is the change in area?

$A = \frac{1}{2}(b)(h)$

$A = [1][2][3]$

$A = 6$ or the area increases by a factor of six

Notice that the $\frac{1}{2}$ in the triangle equation is a constant and that constants are always replaced with a [1] because multiplying a number by one is the same as keeping it constant.

2. The area of a square follows the equation: $A = (length)^2$

If the length of a square's side triples, what happens to the area?

$A = (l)^2$

$A = [3]^2$

$A = 9$ or the area increases by a factor of nine

3. The volume of a sphere follows the equation: $V = \frac{4}{3} \cdot \pi \cdot (radius)^3$

If the volume decreases by a factor of 27, what must have happened to the radius?

$V = \frac{4}{3} \cdot \pi \cdot (r)^3$

$\left[\frac{1}{27}\right] = [1][1] \cdot (r)^3$

$\sqrt[3]{\left[\frac{1}{27}\right]} = \frac{1}{3} = r$ or the radius decreased by a factor of three

Again, the $\frac{4}{3}$ and π are both constants, so are both replaced with a one.

4. Suppose we have a cylindrical rubber cord with a length, L, and a cross-sectional area, A. The volume of the cord is then:

$V = L \cdot A$ or, rearranged, $A = \frac{V}{L}$

We stretch the cord so that the length doubles. What happens to the cross-sectional area if the volume remains fixed?

$$A = \frac{V}{L}$$

$A = \frac{[1]}{[2]}$ or the cord becomes one-half as thick

5. Suppose we take a sample of bubble solution and add it to the end of a straw, using it to blow a bubble. The volume of the sample as it makes the film of the bubble follows the equation:

$V = 4 \cdot \pi \cdot (r)^2 \cdot t$ where *r* is the radius of the bubble and *t* is the thickness of the bubble film

If the bubble expands and the radius doubles, what happens to the thickness of the bubble, assuming the volume of the bubble solution remains constant?

$V = 4 \cdot \pi \cdot (r)^2 \cdot t$

$[1] = [1] \cdot [1] \cdot (2)^2 \cdot t$ so $t = \frac{1}{4}$ or the thickness is one-fourth the previous thickness

We can see this more immediately by rearranging the initial equation to:

$$t = \frac{V}{4 \cdot \pi \cdot (r)^2}$$

Ignoring the constants, the equation shows thickness is inversely proportional to the radius-squared:

$$t \propto \frac{1}{r^2}$$

As the radius increases from 1 to 2 to 3 to 4

the denominator increases from 1 to 4 to 9 to 16

and the value of the whole fraction *decreases* from 1 to $\frac{1}{4}$ to $\frac{1}{9}$ to $\frac{1}{16}$

The thickness of the bubble decreases accordingly.