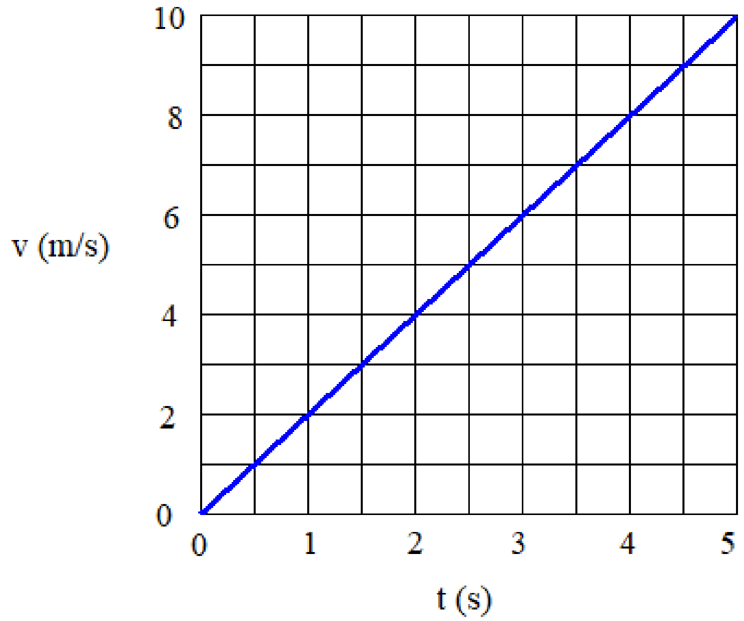
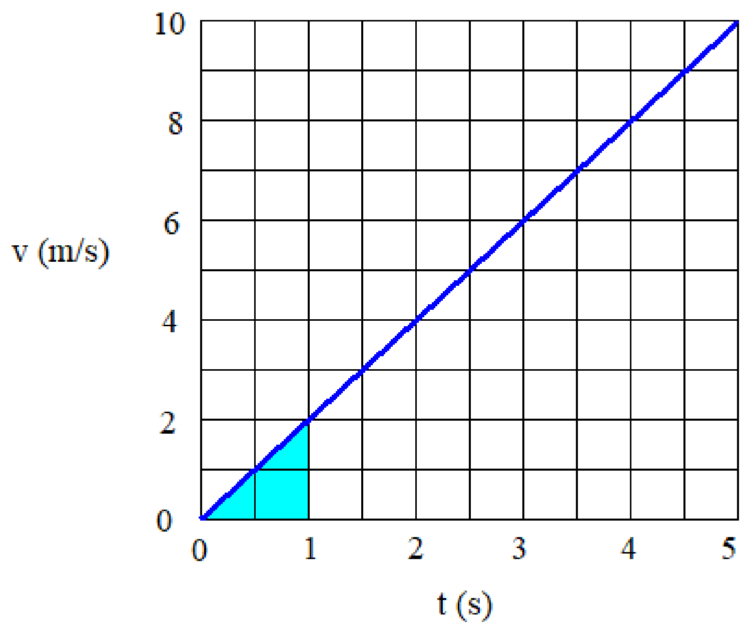
AP Physics 1 – Summer Tutorial 5

Position graphs with acceleration



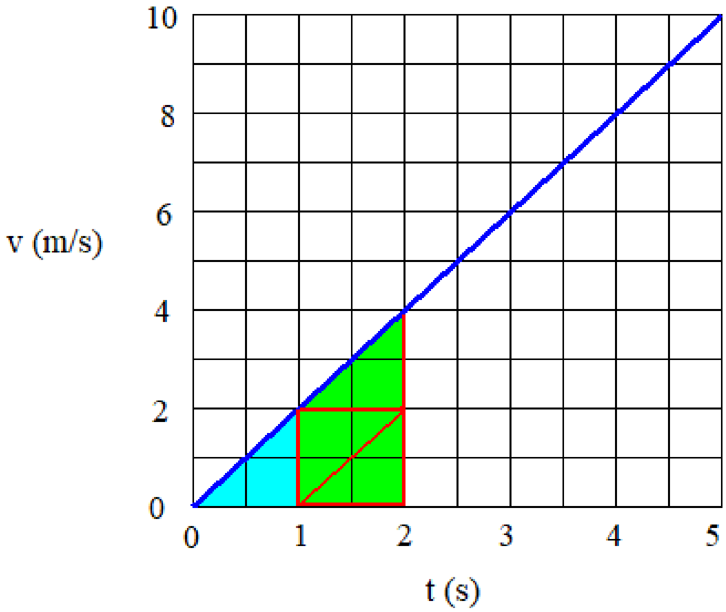
Suppose the graph above shows a car moving with an acceleration of 2 m/s^2. For simplicity, we'll say that the position begins at 0m.

To see what the position versus time graph looks like, we'll use the principle that the area under a velocity versus time graph equals the change in position.

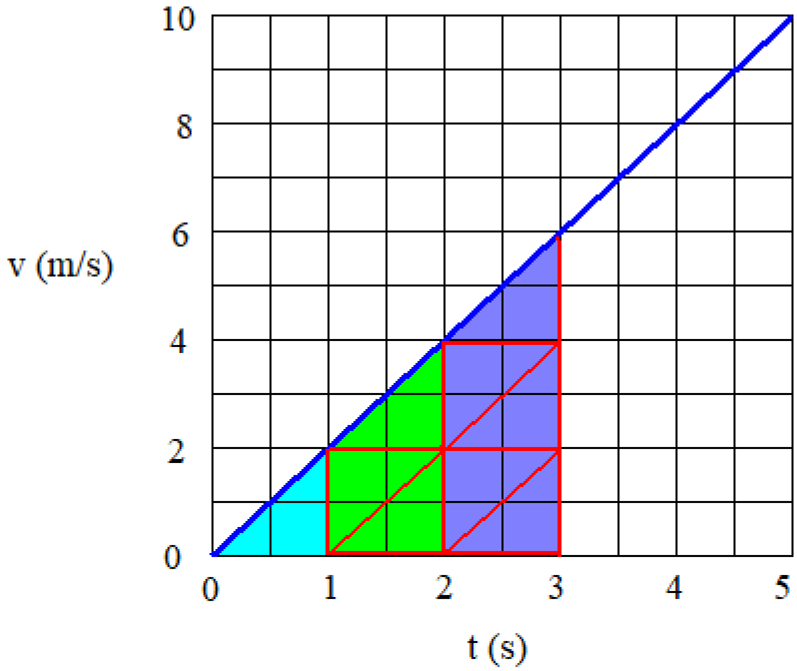


In the first second, the area is the area of the shaded triangle:

$\frac{1}{2}\cdot$(base)$\cdot$(height) = $\frac{1}{2}\cdot$(1s)$\cdot$(2m/s) = 1m



If we shade the area from 1s to 2s, we can see that it's made of three triangles the same size as the triangle between 0s and 1s. If the area from 0s to 1s is 1m, then the area from 1s to 2s is 3m.



The area trapped from 2s to 3s is five such triangles, which is a displacement of 5m.

So a pattern begins to appear:

Time span	Displacement (m)
0s – 1s	1
1s – 2s	3
2s – 3s	5
3s – 4s	7
4s – 5s	9

This makes sense, conceptually. If the object is accelerating from 0m/s to 10m/s, it is speeding-up, so it is gradually traveling greater and greater distances per second of time.

We should also note that these displacements are cumulative.

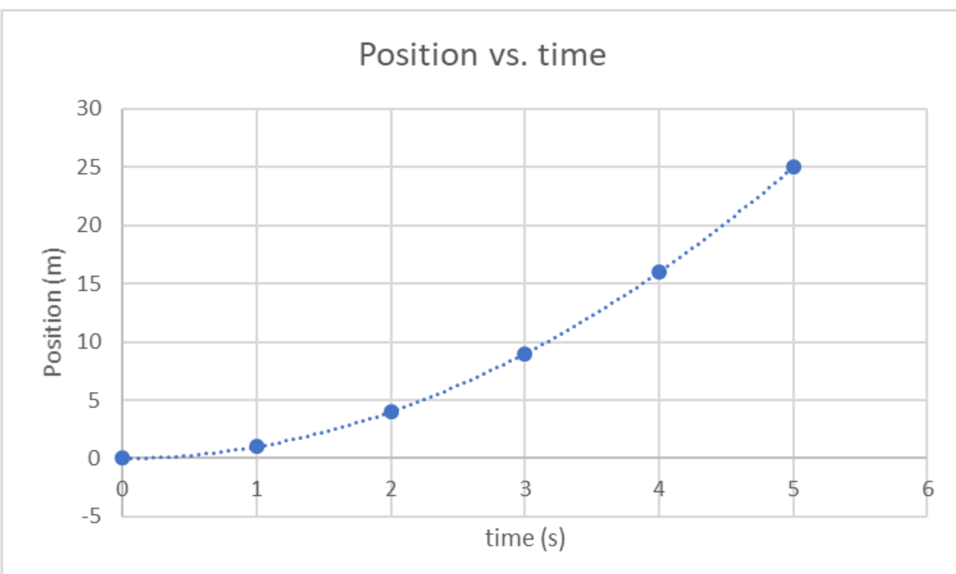
If the car travels 1m from 0s to 1s and 3m from 1s to 2s, it travels a total of 4m from 0s to 2s.

Likewise, 1m (0s to 1s) + 3m (1s to 2s) + 5m (2s to 3s) = a total of 9m from 0s to 3s.

The cumulative chart looks like this:

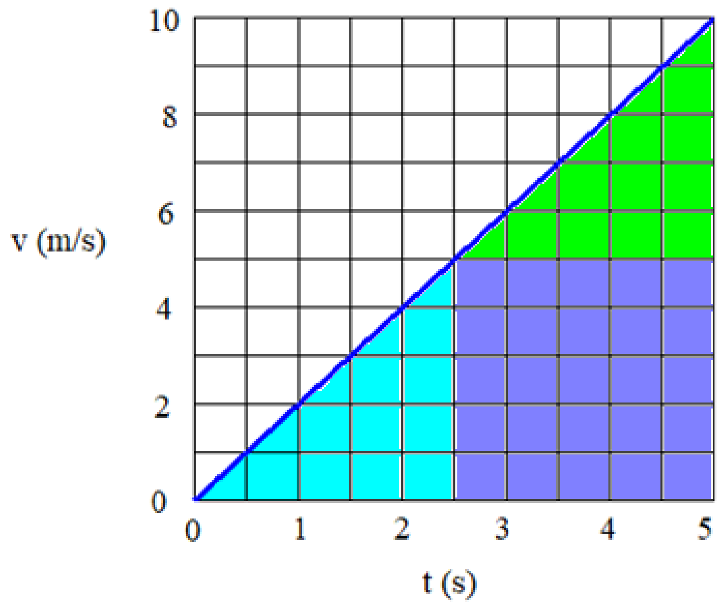
Time (s)	Displacement (m)
0	0
1	1
2	4
3	9
4	16
5	25

which is simply the quadratic function: $s = t^2$

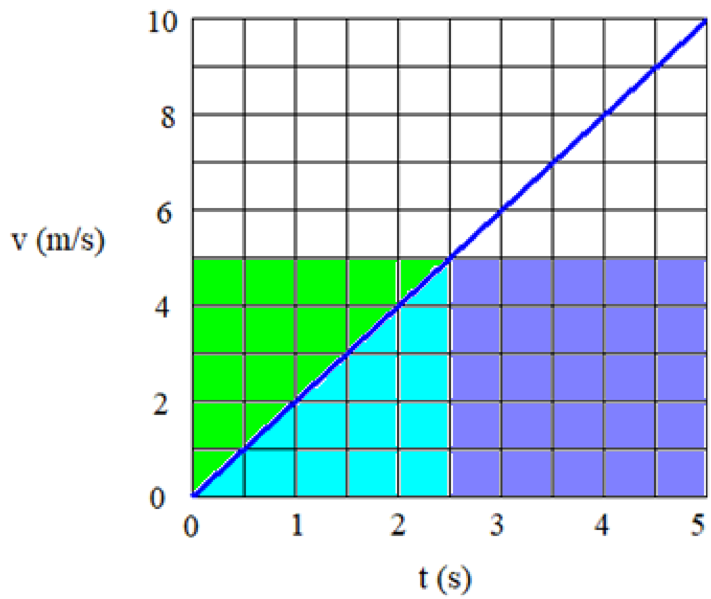


the graph of which is parabolic. We can see in the graph, the slope is positive and gradually increases. This agrees with the velocity graph on the first page, as the velocity value itself is positive and gradually increasing.

We can also shade the graph into three sections



and tip the green area over on-top of the light blue.



The total area here is the total time (5 seconds) multiplied by the midpoint velocity (5m/s which occurs at 2.5s), so the total displacement is 25m.

The midpoint velocity is the arithmetic mean of the initial velocity (0m/s) and the final velocity (10m/s).

We've previously defined average velocity as total displacement divided by total time:

$\overline{v} = \frac{\Delta s}{\Delta t}$ and now we're also stating $\frac{\Delta s}{\Delta t}$ = midpoint velocity = $\frac{v_i + v_f}{2}$

All together, gives us the result that, if acceleration is constant, average velocity, $\overline{v} = \frac{v_i + v_f}{2}$.

This may be intuitive. If you begin making 10 $\frac{dollars}{hour}$ and over one-thousand hours of work, this gradually increases to 20 $\frac{dollars}{hour}$, then it's reasonable that you would earn a total of $15,000.

But $\overline{v} = \frac{v_i+v_f}{2}$ doesn't work if the acceleration is not constant. Your employer can't keep you at 10 dollars/hour for a year, pay you 20 dollars/hour for your last day on the job, and then claim your average wage was 15 dollars/hour.

Here is a summary of important kinematics ideas so far:

1. Velocity is the slope of a position versus time graph

2. If that slope is constant, $v = \frac{\Delta s}{\Delta t}$ for any given time span.

3. If the slope is not constant, then all we may know is the average velocity for any time span, $\overline{v} = \frac{\Delta s}{\Delta t}$.

4. Acceleration is the slope of a velocity versus time graph.

5. If that slope is constant, $a = \frac{\Delta v}{\Delta t}$ for any given time span.

6. Also, if acceleration is constant, $\overline{v} = \frac{v_i+v_f}{2}$.

7. The area trapped to the x-axis of a velocity versus time graph is the displacement or change in position.

8. Likewise, the area trapped to the x-axis of an acceleration versus time graph (though rarely used) would be the change in velocity.