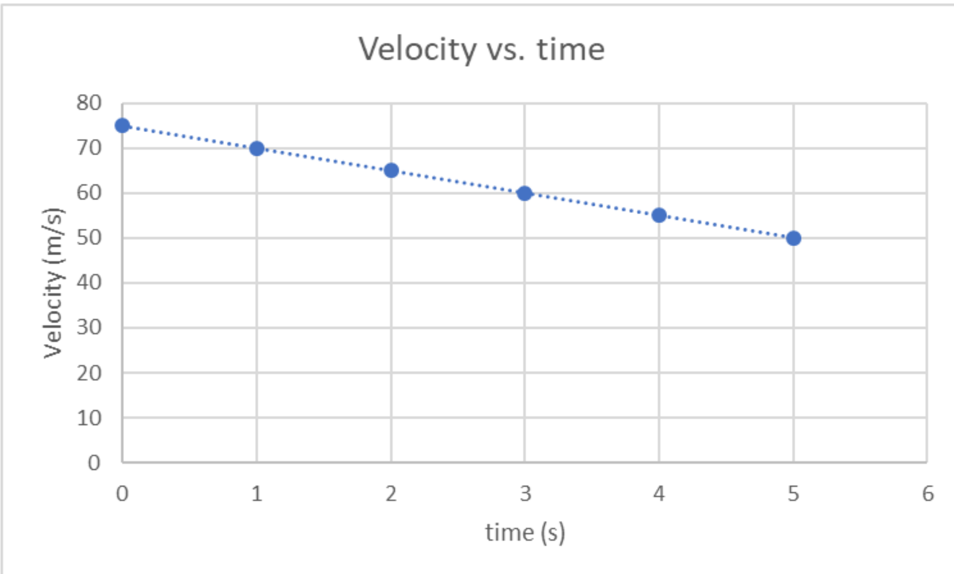
AP Physics 1 – Summer Tutorial 6

Linearization

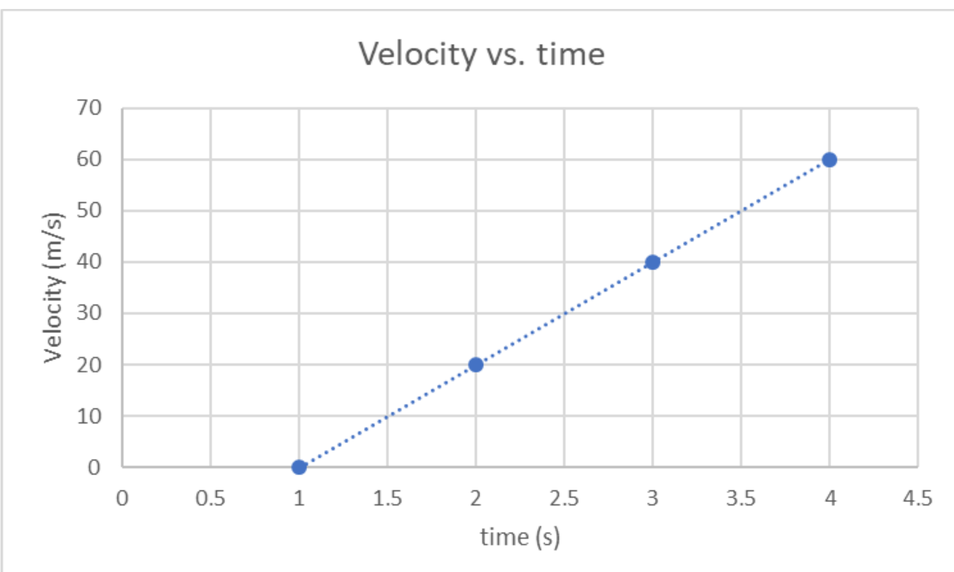
We've seen several linear graphs and found their related functions by determining the slope and y-intercept, and then filling-out the form:

$$y = mx + b$$



For example, the graph above has the function, $v = -5 \cdot t + 75$.

And it's not too difficult even if the function is linear with no immediately obvious y-intercept.

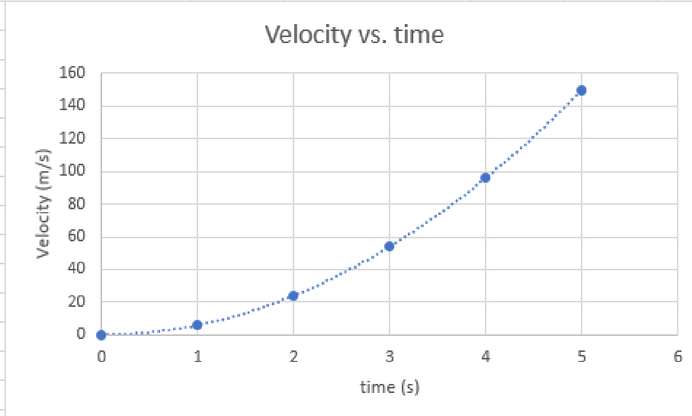


For instance, the graph above has a slope of 20m/s^2 and an *x-intercept* of 1s, but we still need the *y-intercept* to use the form y = mx + b. Well, just solve for it. When y = 0, x = 1, so:

y = mx + b becomes 0 = (20)(1) + b therefore b = -20m/s.

But what if we collect data, make a graph, and the graph is not linear. How can we find the function? Suppose, for instance, we graph the following curve:

Time (s)	Velocity (m/s)
0	0
1	6
2	24
3	54
4	96
5	150

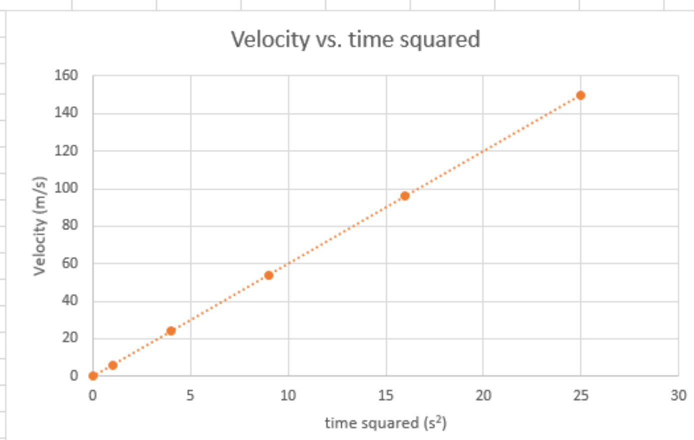


It's clearly not a linear function. Maybe it's quadratic; it does look parabolic. At least, that can be your first guess. It might not be right, but there's no harm in following Occam's razor at the start.

If it is quadratic, how can we find the function of the graph? The trick is to convert the graph into a linear graph, find that linear function, and then translate that linear function back into the function of the original graph. This is how that's done:

Just by inspection, the graph is increasing faster along the y-axis than along the x-axis. We want to "balance" the increases so that both axes increase together, making the graph linear. We do that by squaring the x-axis values so that the x-axis can "catch up" with the more quickly increasing y-axis. Then we make that graph of y-values vs. x-squared values:

Time squared (s^2)	Time (s)	Velocity (m/s)
0	0	0
1	1	6
4	2	24
9	3	54
16	4	96
25	5	150

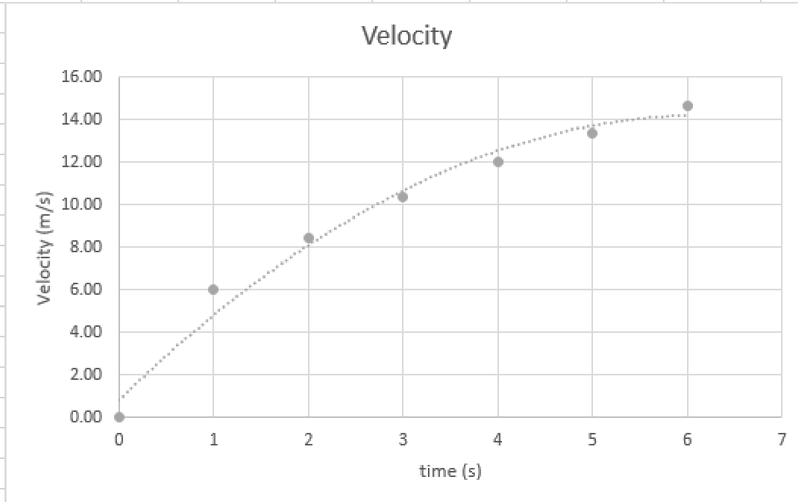


Now this is a linear function we can work with. The slope is 6 and the y-intercept is zero, so:

$v = 6t^2$ and that's it. That's the function of the first graph we were looking for.

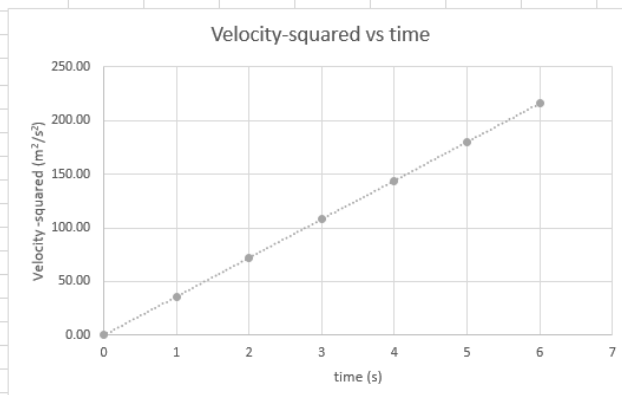
What if we make a graph and it curves the other way?

Time (s)	Velocity (m/s)
0	0.00
1	6.00
2	8.49
3	10.39
4	12.00
5	13.42
6	14.70



As you might guess, we follow a very similar process, but here the x-axis is increasing faster than the y-axis. We need to square all the y-axis values to "catch up".

Time (s)	Velocity (m/s)	Velocity squared (m^2/s^2)
0	0.00	0.00
1	6.00	36.00
2	8.49	72.00
3	10.39	108.00
4	12.00	144.00
5	13.42	180.00
6	14.70	216.00

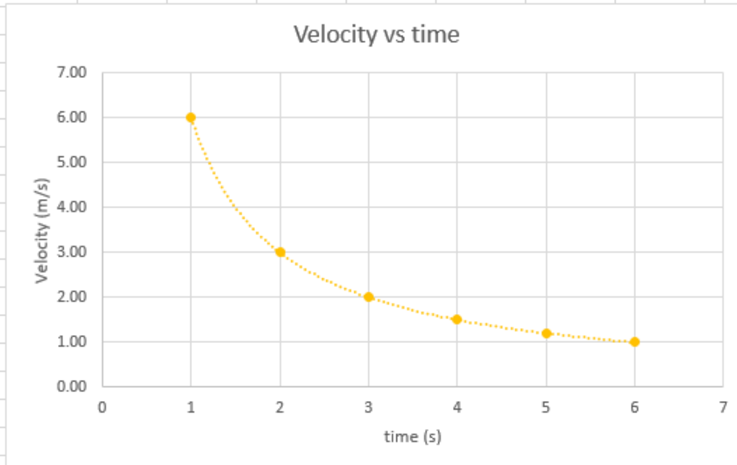


This has a linear function we can find:

$v^2 = 36t$ or $v = 6\sqrt{t}$ as the function of the original data.

One final linearization which is common is when you see a graph that looks like this:

Time (s)	Velocity (m/s)
1	6.00
2	3.00
3	2.00
4	1.50
5	1.20
6	1.00



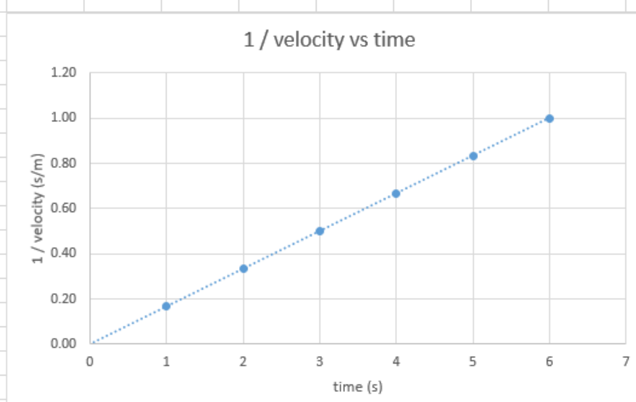
Here, the velocity is *decreasing* as the time increases, which is a new sort of problem. But we've seen something like this before, briefly, in the tutorial on proportional reasoning:

As the denominator of a fraction gradually increases, the value of the whole fraction gradually decreases

as in: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$

So we can make the y-axis increase rather than decrease by graphing $\frac{1}{velocity}$ on the y-axis:

Time (s)	Velocity (m/s)	1 / velocity (s/m)
1	6.00	0.17
2	3.00	0.33
3	2.00	0.50
4	1.50	0.67
5	1.20	0.83
6	1.00	1.00



The graph of this linear function can be found as $\frac{1}{v} = \frac{1}{6} \cdot t$ or, solving: $v = \frac{6}{t}$