HYPERBOLIC SIN: $\sinh x = \frac{e^x - e^{-x}}{2}$

HYPERBOLIC COS: $\cosh x = \frac{e^x + e^{-x}}{2}$

HYPERBOLIC TAN: $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

LET $z = e^x$

$$y = \tanh x = \frac{z - \frac{1}{z}}{z + \frac{1}{z}} \cdot \frac{z}{z} = \frac{z^2 - 1}{z^2 + 1}$$

$$(z^2+1)y = z^2 - 1$$

$$z^2 y + y = z^2 - 1$$

$$z^2 y = z^2 - 1 - y$$

$$z^2 y - z^2 = -y - 1$$

$$z^2(y-1) = -1(y+1)$$

$$z^2 = \frac{1+y}{1-y}$$

$$z^2 = e^{2x}$$

$$\ln z^2 = 2x$$

$$x = \frac{1}{2}\ln z^2 = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

$$\tanh^{-1}(y) = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

$$\frac{d}{dy}\tanh^{-1}(y) = \frac{d}{dy}\left[\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)\right] = \frac{1}{1-y^2}$$

$$\text{so } \int \frac{1}{1-y^2}\,dy = \tanh^{-1}(y)$$

TAKE DOWN TO BE POSITIVE

$$F = mg - bv^2$$

$$ma = mg - bv^2$$

$$a = g - \frac{b}{m}v^2$$

$$\frac{dv}{dt} = g - \frac{b}{m}v^2$$

$$\frac{1}{g - \frac{b}{m}v^2}\,dv = dt$$

$$\int \frac{1}{g - \frac{b}{m}v^2}\,dv = \int_0^t dt$$

$$\int \frac{\frac{1}{g}}{1 - \frac{b}{mg}v^2}\,dv = t$$

$$\frac{1}{g}\int \frac{1}{1 - \frac{b}{mg}v^2}\,dv = t$$

$$\int \frac{1}{1 - \frac{b}{mg}v^2}\,dv = gt$$

LET $u^2 = \frac{b}{mg}v^2$

$$u = \sqrt{\frac{b}{mg}}\,v$$

$$\frac{du}{dv} = \sqrt{\frac{b}{mg}}$$

$$dv = \sqrt{\frac{mg}{b}}\,du$$

$$\sqrt{\frac{mg}{b}}\int \frac{1}{1-u^2}\,du = gt$$

$$\sqrt{\frac{mg}{b}}\tanh^{-1}u = gt$$

$$\tanh^{-1}u = \frac{gt}{\sqrt{\frac{mg}{b}}}$$

$$u = \tanh\left[\frac{gt}{\sqrt{\frac{mg}{b}}}\right]$$

$$v\sqrt{\frac{b}{mg}} = \tanh\left[\frac{gt}{\sqrt{\frac{mg}{b}}}\right]$$

$$\boxed{v = \sqrt{\frac{mg}{b}}\tanh\left[\sqrt{\frac{gb}{m}}\,t\right]}$$

$$a = \frac{dv}{dt} = \sqrt{\frac{mg}{b}}\cdot\sqrt{\frac{gb}{m}}\left[1 - \tanh^2\sqrt{\frac{gb}{m}}\,t\right]$$

$$\boxed{a = g\left[1 - \tanh^2\sqrt{\frac{gb}{m}}\,t\right]}$$