PROBLEM SET 16

1. A BOX WITH WIDTH 5, LENGTH 4, AND HEIGHT 3.

$$\int_{-2}^{2}\int_{1}^{6} 3 \cdot dy \cdot dx = \int_{-2}^{2} \left[3y\right]_{1}^{6} dx =$$

$$\int_{-2}^{2} 15 \cdot dx = 15x\Big]_{-2}^{2} = 60$$

2. $$\int_{0}^{5} 12x^2y^3\,dx = 4x^3y^3\Big]_{0}^{5} = 500y^3$$

$$\int_{0}^{1} 12x^2y^3\,dy = 3x^2y^4\Big]_{0}^{1} = 3x^2$$

3. $$\int_{1}^{4}\int_{0}^{2} 6x^2y - 2x \cdot dy \cdot dx =$$

$$\int_{1}^{4} \left[3x^2y^2 - 2xy\right]_{0}^{2} dx =$$

$$\int_{1}^{4} 12x^2 - 4x \cdot dx =$$

$$4x^3 - 2x^2\Big]_{1}^{4} = 224 - 2 = 222$$

4. $$\int_0^2 \int_0^4 y^3 \cdot e^{2x} \cdot dy \cdot dx =$$

$$\int_0^2 \left[\tfrac{1}{4} y^4 \cdot e^{2x} \right]_0^4 dx =$$

$$\int_0^2 64 e^{2x} \, dx =$$

$$32 e^{2x} \Big]_0^2 = 32 \left(e^4 - 1 \right)$$

5. $$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x-y) \, dy \cdot dx =$$

$$\int_0^{\pi/2} \left[\cos(x-y) \right]_0^{\pi/2} dx =$$

$$\int_0^{\pi/2} \cos\left(x - \tfrac{\pi}{2}\right) - \cos(x) \, dx =$$

$$\sin\left(x - \tfrac{\pi}{2}\right) - \sin(x) \Big]_0^{\pi/2} =$$

$$\left[\sin(0) - \sin\left(\tfrac{\pi}{2}\right) \right] - \left[\sin\left(\tfrac{-\pi}{2}\right) - \sin(0) \right] = 0$$

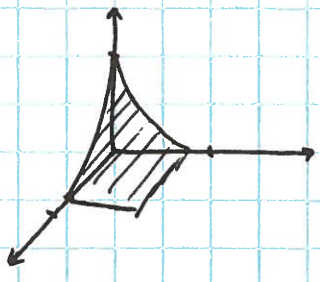
6. $$\int_0^2 \int_1^2 \left(y + x \cdot y^{-2}\right) \cdot dy \cdot dx =$$

$$\int_0^2 \left[\frac{1}{2}y^2 - xy^{-1}\right]_1^2 dx =$$

$$\int_0^2 \frac{3}{2} + \frac{x}{2}\, dx =$$

$$\frac{3}{2}x + \frac{x^2}{4}\Big]_0^2 = 4$$

7.



8. $$z = 2x + 3y + \frac{15}{2}$$

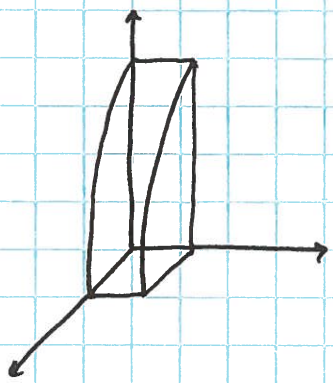
$$\int_{-1}^2 \int_{-1}^1 2x + 3y + \frac{15}{2}\, dy \cdot dx =$$

$$\int_{-1}^2 \left[2xy + \frac{3}{2}y^2 + \frac{15}{2}y\right]_{-1}^1 dx =$$

$$\int_{-1}^2 4x + 15 \cdot dx = 2x^2 + 15x\Big]_{-1}^2 =$$

$$38 - (-13) = 51$$

9.



$$\int_0^4 \int_0^5 (16 - x^2)\, dy \cdot dx =$$

$$\int_0^4 \left[16y - x^2 y\right]_0^5 dx =$$

$$\int_0^4 80 - 5x^2\, dx =$$

$$80x - \frac{5}{3}x^3 \Big]_0^4 = \frac{640}{3}$$

10. $$V = \int_{-1}^{1} \int_0^5 x^2 y \cdot dy \cdot dx =$$

$$\int_{-1}^{1} \left[\frac{x^2 y^2}{2}\right]_0^5 dx =$$

$$\int_{-1}^{1} \frac{25}{2} x^2\, dx =$$

$$\frac{25}{6} x^3 \Big]_{-1}^{1} = \frac{50}{6}$$

$$\bar{z} = \frac{V}{A} = \frac{50/6}{10} = \frac{5}{6}$$