Problem Set 13

1. For the functions
$z = \cos(x + 4y)$
$x = 5t^4$
$y = \frac{1}{t}$
use the chain rule to find $\frac{dz}{dt}$.

2. For the functions
$z = x^2y^3$
$x = s\cdot\cos(t)$
$y = 1 - 2\cdot s\cdot t$
use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

3. Use a tree diagram to write-out the chain rule for w = f(r,s,t), given:
r = r(x,y)
s = s(x,y)
t = t(x,y)

4. Use the chain rule to find the partial derivatives $\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ for
$z = x^4 + x^2y$
$x = s + 2t - u$
$y = s\cdot t\cdot u^2$
when s = 4, t = 2, and u = 1

5. Use the chain rule to find the partial derivatives $\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}$ for
$P = \sqrt{u^2 + v^2 + w^2}$
$u = x\cdot e^y$
$v = y\cdot e^x$
$w = e^{xy}$
when x = 0, y = 2

6. Use $\frac{dy}{dx} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$ for the equation $y\cdot\cos(x) = x^2 + y^2$

7. Use $\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$ and $\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$ to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the equation $x^2 + 2y^2 + 3z^2 = 1$.

8. The radius of a right circular cone is increasing at a rate of 1.8cm/s while its height is decreasing at a rate of 2.5cm/s. At what rate is the volume of the cone changing when the radius is 120cm and the height is 140cm?

9. The voltage, V, in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance, R, is slowly increasing as the resistor heats up. Use Ohm's law, V = IR, to find how the current, I, is changing at the moment when R = 400Ω, I = 0.08A, $\frac{dV}{dt}$ = -0.01V/s, and $\frac{dR}{dt}$ = 0.03Ω/s.

10. If z = f(x,y) where x = r·cos(θ) and y = r·sin(θ), find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$, then show that $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}\cdot(\frac{\partial z}{\partial \theta})^2$.