Problem Set 15

1. Find the local maximum and minimum values and saddle point(s) of the function $f(x,y) = x^3 - 12xy + 8y^3$.

2. Find the local maximum and minimum values and saddle point(s) of the function $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.

3. Find the absolute maximum and minimum values of $f(x,y) = x^2 + y^2 + x^2y + 4$ in the region $D = \{(x,y) \mid |x| \le 1, |y| \le 1\}$.

4. Find the shortest distance from the point (2, 0, -3) to the plane $x + y + z = 1$.

5. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant *c*.

6. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = y^2 - x^2$ given the constraint $\frac{1}{4}x^2 + y^2 = 1$.

7. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = e^{xy}$ given the constraint $x^3 + y^3 = 16$.

8. Find the extreme values of the function $f(x,y,z) = x + 2y$ given the constraints $x + y + z = 1$ and $y^2 + z^2 = 4$.

9. Find the extreme values of the function $f(x,y) = x^2 + y^2 + 4x - 4y$ within the region of $x^2 + y^2 \le 9$.

10. Use Lagrange multipliers to find three positive numbers whose sum is 100 and whose product is a maximum.