Problem Set 19

1. Evaluate the iterated integral $\int_0^1 \int_x^{2x} \int_0^y (2xyz)dz \cdot dy \cdot dx$

2. Use a triple integral to find the volume of the tetrahedron enclosed by the coordinate planes and the plane 2x + y + z = 4.

3. Sketch the solid whose volume is given by the iterated integral
$\int_0^1 \int_0^{1-x} \int_0^{2-2z} dy \cdot dz \cdot dx$

4. Express the integral $\int\int\int f(x, y, z) \cdot dV$ as an iterated integral six different ways for the solid bounded by the surfaces $y = 4 – x^2 – 4z^2$ and y = 0.

5a. Convert the cylindrical coordinates $(\sqrt{2}, \frac{3\pi}{4}, 2)$ to rectangular coordinates.
5b. Convert the rectangular coordinates $(2\sqrt{3}, 2, -1)$ to cylindrical coordinates.

6. Write the equation $x^2 – x + y^2 + z^2 = 1$ in cylindrical coordinates.

7. Evaluate $\iiint z \cdot dV$ for the volume enclosed by the paraboloid $z = x^2 + y^2$ and the plane z = 4.

8a. Convert the spherical coordinates $(2, \frac{\pi}{4}, \frac{\pi}{4})$ to rectangular coordinates.
8b. Convert the rectangular coordinates $(1, 0, \sqrt{3})$ to spherical coordinates.

9. Write the equation $x^2 – 2x + y^2 + z^2 = 0$ in spherical coordinates.

10. Use spherical coordinates to evaluate $\iiint (x^2 + y^2 + z^2)dV$ for a sphere centered at the origin with a radius of 5.