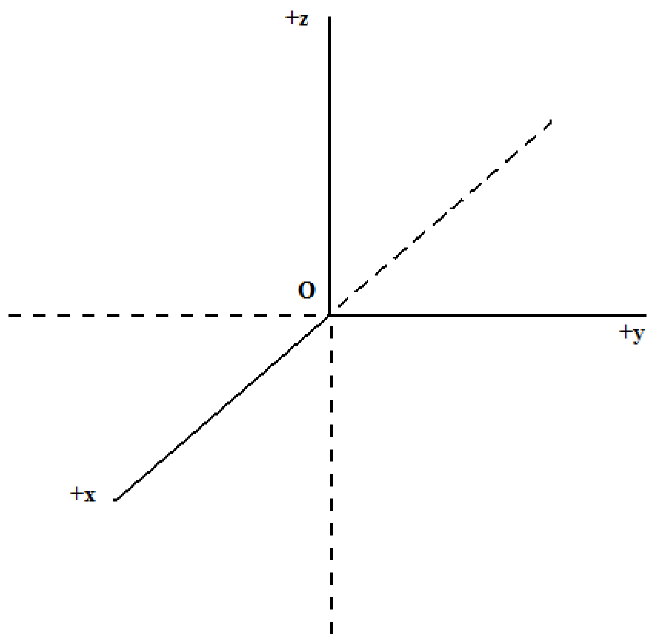
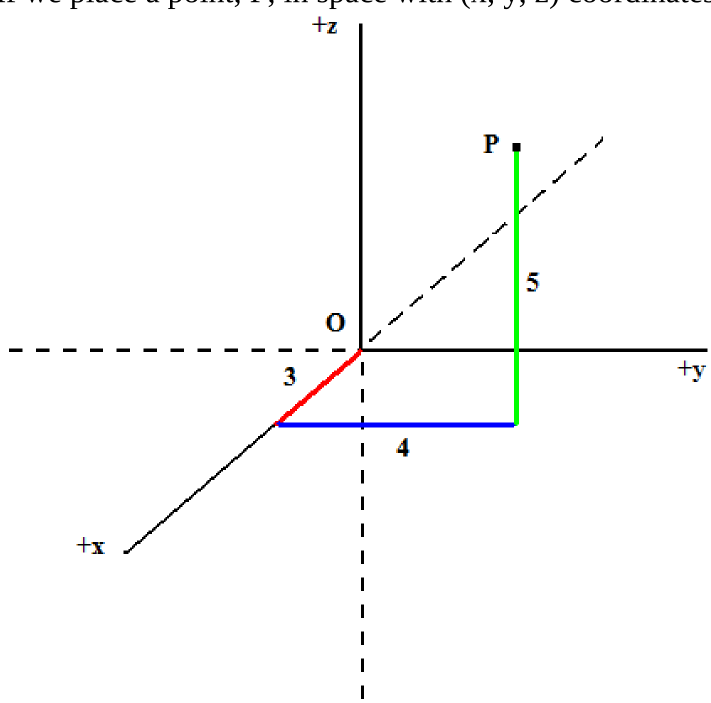
3D Coordinate Systems and Vectors

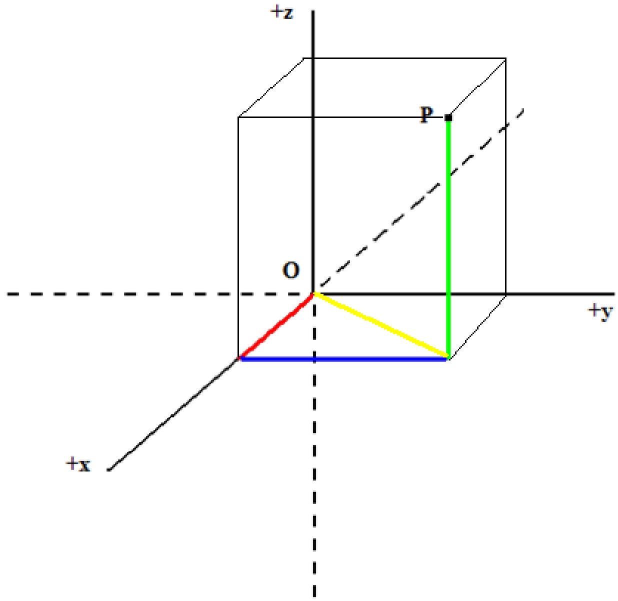


Above is a standard diagram of the three coordinate axes centered at the origin and which contains three coordinate planes, the xz plane (consisting of all points where y = 0), the xy plane (where z = 0), and the yz plane (where x = 0). These three planes divide space into eight octants.

If we place a point, P, in space with (x, y, z) coordinates of (3, 4, 5), it would look like this:



We can imagine point P sitting on one corner of a box with the origin at the opposite corner:



Let the red line have a length P_x, the blue line have a length P_y, and the green line have a length P_z.

From the Pythagorean theorem, the yellow line must have a length $\sqrt{P_x^2 + P_y^2}$.

The distance between the origin and P must then be $\sqrt{P_z^2 + \left(\sqrt{P_x^2 + P_y^2}\right)^2} = \sqrt{P_x^2 + P_y^2 + P_z^2}$

In general, the distance between any two points in space is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

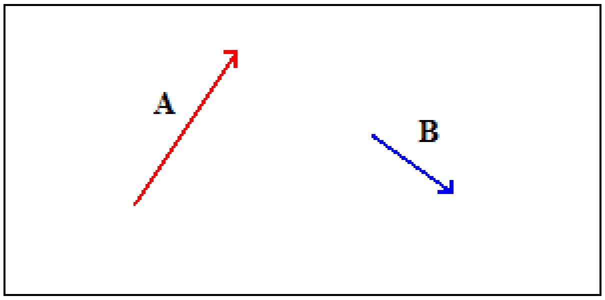
If we imagine all points in space which satisfy the equation $x^2 + y^2 + z^2 = R^2$, they would all be points a distance R from the origin, making a spherical surface with radius R centered at the origin.

In one dimension, an equation (like $x^2 = 5$) is a point.
In two dimensions, an equation (like $x^2 + y^2 = 5$) is a curve.
In three dimensions, an equation (like $x^2 + y^2 + z^2 = 5$) is a surface.

To shift the sphere right, up, or out of the page, we could generalize to the equation:

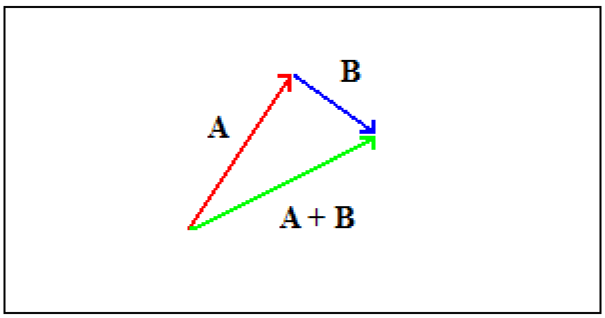
$(x - h)^2 + (y - j)^2 + (z - l)^2 = R^2$

A vector can be defined as a quantity with magnitude and direction. It is generally represented with an arrow where the length of the arrow is proportional to the magnitude of the vector.

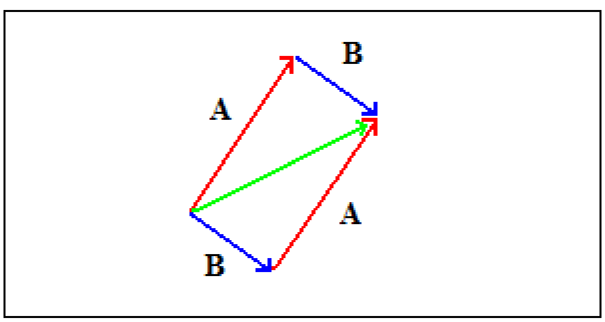


Vectors can be moved and (as long as they maintain their length and direction), they maintain their identity.

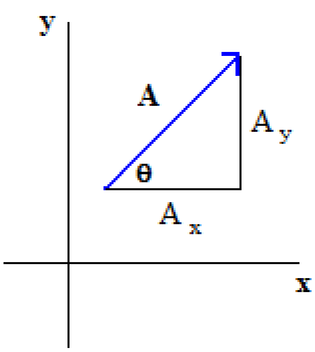
To add vectors, simply align them “tail-to-tip” and draw the resultant summation:



You can also draw a parallelogram so that the vector sum crosses the parallelogram from corner to corner, showing that **A** + **B** = **B** + **A**:



Often it is useful to find the components of a vector, most commonly the components parallel to the principle axes.



In the diagram above, vector **A** can be written as (A_x, A_y) where

$A_x = |A|\cdot\cos\theta$
$A_y = |A|\cdot\sin\theta$

Where $|A|$ is the magnitude of the vector

Vector addition can then be computed as:

$\mathbf{A} + \mathbf{B} = (A_x, A_y) + (B_x, B_y) = (A_x+B_x, A_y+B_y)$

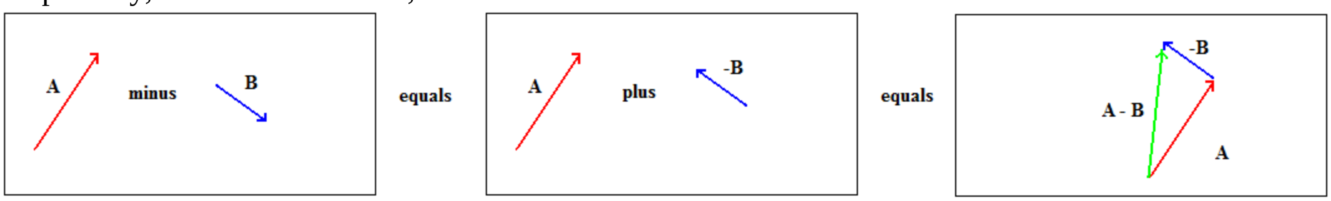
This notation also makes clear the result of vector subtraction:

$\mathbf{A} - \mathbf{B} = (A_x, A_y) - (B_x, B_y) = (A_x - B_x, A_y - B_y)$

which is the same as

$\mathbf{A} + (-\mathbf{B}) = (A_x, A_y) + (-B_x, -B_y) = (A_x - B_x, A_y - B_y)$

Graphically, **-B** is the same as **B**, rotated 180º



A scalar is a quantity with magnitude only. A scalar (c) can be multiplied by a vector (**A**) to produce a second vector:

$c\mathbf{A} = c\cdot(A_x, A_y) = (cA_x, cA_y)$

The following is a general list of vector properties, assuming vectors **A**, **B**, and **C** and scalars c and d.

1. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

2. $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$

3. $\mathbf{A} + 0 = \mathbf{A}$

4. $\mathbf{A} + (-\mathbf{A}) = 0$

5. $c(\mathbf{A} + \mathbf{B}) = c\mathbf{A} + c\mathbf{B}$

6. $(c + d)\mathbf{A} = c\mathbf{A} + d\mathbf{A}$

7. (cd)**A** = c(d**A**)

8. 1·**A** = **A**

These are all very easy to prove using vector components.

Lastly, we will define the unit vectors **i**, **j**, and **k**.

i = (1, 0, 0)
j = (0, 1, 0)
k = (0, 0, 1)

Thus, we can write a vector with unit vector notation. Suppose **A** = (10, 6, -8).

Using unit vectors, we can also write **A** = 10**i** + 6**j** – 8**k**.