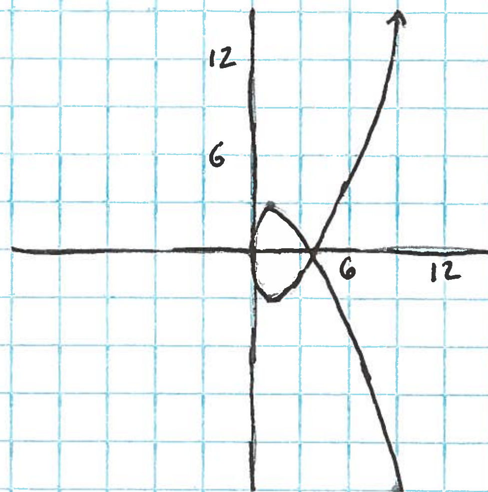
PROBLEM SET 1

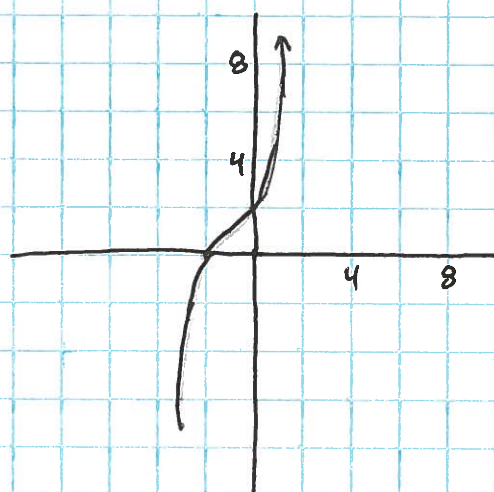
1.

t	x	y
-3	9	-15
-2	4	0
-1	1	3
0	0	0
1	1	-3
2	4	0
3	9	15



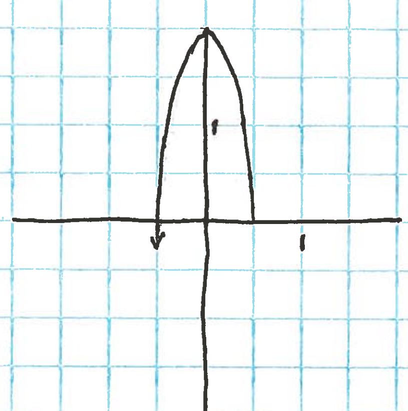
2.

t	x	y
-2	-3	-7
-1	-2	0
0	-1	1
1	0	2
2	1	9

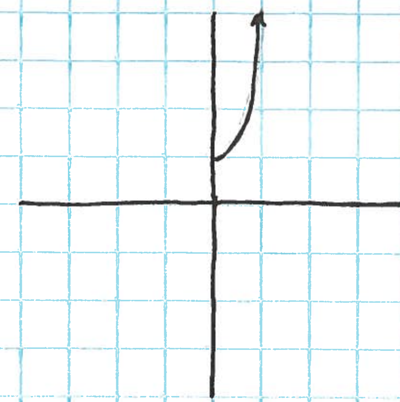


3.

$$\cos^2\theta + \sin^2\theta = 4x + \frac{y^2}{4} = 1$$



4. $y = e^{2t} = (e^t)^2 = (x+1)^2$



5. $x = t^{-1}$

$y = t^{\frac{1}{2}} \cdot e^{-t}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = e^{-t}\left(t^{5/2} - \tfrac{1}{2}t^{3/2}\right)$$

6. $\text{SLOPE} = \dfrac{dy/dt}{dx/dt} = \dfrac{-3t^2}{4-2t} = \dfrac{-3}{2}$ AT $t=1$

AT $t=1$, $x=4$ AND $y=1$, SO $1 = -\frac{3}{2}\cdot 4 + b$

$b = 7$ SO $y = -\frac{3}{2}x + 7$

7. $x = t^2 + 1$ $\qquad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{e^t}{2t}$

$y = e^t - 1$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{1}{2}e^t \cdot t^{-1}\right)}{2t}$$

$$= \frac{e^t}{4t^2}\left(1 - \frac{1}{t}\right)$$ POSITIVE WHEN $t > 1$.

8. $x = t^3 - 3t$ $\qquad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t}{3t^2-3}$

$y = t^2 - 3$

a. $\dfrac{dy}{dx} = 0$ IF $t = 0$ AT $(0, -3)$

b. $\dfrac{dy}{dx} = \pm\infty$ IF $t = \pm 1$ AT $(-2, -2)$ AND $(2, -2)$

c. $\dfrac{dy}{dx} = 1 = \dfrac{2t}{3t^2-3}$ OR $3t^2 - 2t - 3 = 0$

$t = \dfrac{1+\sqrt{10}}{3}$ AT $(-1.5, -1.1)$

9. $$A = \int_{t_1}^{t_2} y(t) \cdot \frac{dx}{dt} \cdot dt$$

$$= \int_0^2 t^{\frac{1}{2}}(2t-2)\,dt = \frac{4}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}}\Big]_0^2 = 1.288$$

10. $$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$$= \int_0^1 \sqrt{(6t)^2 + (6t^2)^2} \cdot dt$$

$$= \int_0^1 6t\sqrt{1+6t}\,dt = 2\left(1+t^2\right)^{\frac{3}{2}}\Big]_0^1$$

$$= 4\sqrt{2} - 2$$