PROBLEM SET 11

1. $\frac{\partial f}{\partial x} = -\pi e^{-t} \cdot \sin(\pi x)$

$\frac{\partial f}{\partial t} = -e^{-t} \cdot \cos(\pi x)$

2. $\frac{\partial f}{\partial x} = \sin(y-z)$

$\frac{\partial f}{\partial y} = x \cdot \cos(y-z)$

$\frac{\partial f}{\partial z} = -x \cdot \cos(y-z)$

3. $\frac{\partial f}{\partial y} = y \cdot \left[-1(x+y+z)^{-2}\right] + (x+y+z)^{-1}$

$= \frac{-y}{(x+y+z)^2} + \frac{1}{x+y+z} = \frac{-1}{2^2} + \frac{1}{2} = \frac{1}{4}$

4. $\frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial x} 2y^2 + \frac{\partial z}{\partial x} \cdot \frac{\partial}{\partial z} 3z^2 = \frac{\partial}{\partial x}(1)$

$2x + 0 + \frac{\partial z}{\partial x} 6z = 0 \qquad \frac{\partial z}{\partial x} = \frac{-x}{3z}$

$\frac{\partial}{\partial y} x^2 + \frac{\partial}{\partial y} 2y^2 + \frac{\partial z}{\partial y} \frac{\partial}{\partial z} 3z^2 = \frac{\partial}{\partial y}(1)$

$0 + 4y + \frac{\partial z}{\partial y} 6z = 0 \qquad \frac{\partial z}{\partial y} = \frac{-2y}{3z}$

5. $f_x = 3x^2y^5 + 8x^3y$

$f_y = 5x^3y^4 + 2x^4$

$f_{xx} = 6xy^5 + 24x^2y$

$f_{yy} = 20x^3y^3$

$f_{xy} = f_{yx} = 15x^2y^4 + 8x^3$

6. $u_x = 4x^3y^3$ $\qquad u_{xy} = 12x^3y^2$

$u_y = 3x^4y^2 - 4y^3$ $\qquad u_{yx} = 12x^3y^2$

7. $f_x = 4x^3y^2 - 3x^2y$ $\qquad f_{xy} = 8x^3y - 3x^2$

$f_{xx} = 12x^2y^2 - 6xy$ $\qquad f_{xyx} = 24x^2y - 6x$

$f_{xxx} = 24xy^2 - 6y$

8. $u_x = 3x^2 + 3y^2$ $\qquad u_y = 6xy$

$u_{xx} = 6x$ $\qquad u_{yy} = 6x$

$u_{xx} + u_{yy} \neq 0$ so NO

9. $u_t = -6a(x-at)^5 + 6a(x+at)^5$

$u_{tt} = 30a^2(x-at)^4 + 30a^2(x+at)^4$

$u_x = 6(x-at)^5 + 6(x+at)^5$

$u_{xx} = 30(x-at)^4 + 30(x+at)^4$

$30a^2(x-at)^4 + 30a^2(x+at)^4 = a^2\left[30(x-at)^4 + 30(x+at)^4\right]$

10. $\frac{2}{2R_1}\left(\frac{1}{R}\right) = \frac{2}{2R_1}\left(\frac{1}{R_1}\right)$

$\frac{2R}{2R_1} \cdot \frac{2}{2R}\left(R^{-1}\right) = \frac{2}{2R_1}\left(R_1^{-1}\right)$

$\frac{2R}{2R_1}\left(\frac{-1}{R^2}\right) = \frac{-1}{R_1^2}$

$\frac{2R}{2R_1} = \frac{R^2}{R_1^2}$