PROBLEM SET 12

1. $\frac{\partial z}{\partial x} = -4x + 1 = -4(2) + 1 = -7$

$\frac{\partial z}{\partial y} = 6y = 6(-1) = -6$

$z + 3 = -7(x-2) - 6(y+1)$

$7x + 6y + z - 5 = 0$

2. $\frac{\partial z}{\partial x} = xye^{xy} + e^{xy} = 0 + 1 = 1$

$\frac{\partial z}{\partial y} = x^2 e^{xy} = 4 \cdot 1 = 4$

$z - 2 = (x-2) + 4(y-0)$

$x + 4y - z = 0$

3. $\frac{\partial f}{\partial x} = 3x^2y^4 \qquad \frac{\partial f}{\partial y} = 4x^3y^3$

BOTH ARE CONTINUOUS AT $(1,1)$

$\frac{\partial z}{\partial x} = 3x^2y^4 = 3(1^2)(1^4) = 3$

$\frac{\partial z}{\partial y} = 4x^3y^3 = 4(1^3)(1^3) = 4$

$L(x,y) = z_0 + \frac{\partial z}{\partial x}(x - x_0) + \frac{\partial z}{\partial y}(y - y_0)$

$= 1 + 3(x-1) + 4(y-1) = 3x + 4y - 6$

4. $\frac{\partial f}{\partial x} = \frac{xy}{xy-5} + \ln(xy-5)$

$\frac{\partial f}{\partial y} = \frac{x^2}{xy-5}$

BOTH ARE CONTINUOUS AT $(2,3)$

$L = z_0 + \frac{\partial z}{\partial x}(x-x_0) + \frac{\partial z}{\partial y}(y-y_0)$

$= 1 + 6(x-2) + 4(y-3) = 6x + 4y - 23$

5. $dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial t} \cdot dt$

$= -2e^{-2x} \cdot \cos(2\pi t)\,dx - 2\pi \cdot \sin(2\pi t)\,dt$

6. $dm = \frac{\partial m}{\partial p} \cdot dp + \frac{\partial m}{\partial q} \cdot dq$

$= 5p^4q^3\,dp + 3p^5q^2\,dq$

7. $dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$

$= 10x \cdot dx + 2y \cdot dy$

$= 10(1)(0.05) + 2(2)(0.1) = 0.9$

$\Delta z = [5(1.05)^2 + (2.1)^2] - [5(1)^2 + (2)^2] = 0.9225$

8. $A = \ell \cdot w$

$$dA = \frac{\partial A}{\partial \ell} \cdot d\ell + \frac{\partial A}{\partial w} \cdot dw$$

$$= w \cdot d\ell + \ell \cdot dw$$

$$= (24)(0.1) + (30)(0.1) = 5.4\ cm^2$$

9. $dV = 2\left[\pi r^2 \cdot dh\right] + h \cdot 2\pi r \cdot dR$

$$= 2\pi(2^2)(0.1) + (10)(2\pi)(2)(0.05)$$

$$= 2.8\pi$$

10. $P = 8.31 \cdot T \cdot V^{-1}$

$$dP = \frac{\partial P}{\partial T} \cdot dT + \frac{\partial P}{\partial V} \cdot dV$$

$$= \frac{8.31}{V} \cdot dT + \frac{-8.31T}{V^2}\, dV$$

$$= \frac{8.31}{12} \cdot (-5) + \frac{-8.31\,(310)}{12^2}$$

$$= -8.83\ kPa$$