PROBLEM SET 13

1. $$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= -\sin(x+4y) \cdot 20t^3 + -4\sin(x+4y) \cdot (-1 \cdot t^{-2})$$

$$= -20t^3 \sin(x+4y) + \frac{4\sin(x+4y)}{t^2}$$

2. $$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$= 2xy^3 \cdot [-s \cdot \sin(t)] + 3x^2y^2 \cdot [-2s]$$

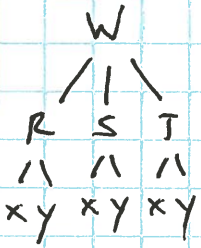
$$= -2sxy^3 \sin(t) - 6sx^2y^2$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= 2xy^3 \cdot \cos(t) + 3x^2y^2 \cdot [-2t]$$

$$= 2xy^3 \cos(t) - 6x^2y^2 t$$

3.



$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial R} \cdot \frac{\partial R}{\partial x} + \frac{\partial w}{\partial s} \cdot \frac{\partial s}{\partial x} + \frac{\partial w}{\partial T} \cdot \frac{\partial T}{\partial x}$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial R} \cdot \frac{\partial R}{\partial y} + \frac{\partial w}{\partial s} \cdot \frac{\partial s}{\partial y} + \frac{\partial W}{\partial T} \cdot \frac{\partial T}{\partial y}$$

4. $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= (4x^3+2xy)(1) + (x^2)(tu^2)$$

$$= 1582$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= (4x^3+2xy)(2) + (x^2)(s\cdot u^2)$$

$$= 3164$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= (4x^3+2xy)(-1) + (x^2)(2stu)$$

$$= -700$$

5. $$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial P}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial P}{\partial w}\cdot\frac{\partial w}{\partial x}$$

$$= \frac{u\cdot e^y}{\sqrt{u^2+v^2+w^2}} + \frac{v\cdot ye^x}{\sqrt{u^2+v^2+w^2}} + \frac{w\cdot ye^{xy}}{\sqrt{u^2+v^2+w^2}}$$

$$= 0 + \frac{4}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{6}{\sqrt{5}}$$

$$\frac{\partial P}{\partial y} = \frac{\partial P}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial P}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial P}{\partial w}\cdot\frac{\partial w}{\partial y}$$

$$= \frac{u\cdot xe^y}{\sqrt{u^2+v^2+w^2}} + \frac{v\cdot e^x}{\sqrt{u^2+v^2+w^2}} + \frac{w\cdot ye^{xy}}{\sqrt{u^2+v^2+w^2}} = \frac{4}{\sqrt{5}}$$

6. $F(x,y) = x^2 + y^2 - y\cos(x) = 0$

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = \frac{2x + y\sin(x)}{\cos(x) - 2y}$$

7. $F(x,y,z) = x^2 + 2y^2 + 3z^2 - 1 = 0$

$$\frac{\partial z}{\partial x} = \frac{-x}{3z} \qquad \frac{\partial z}{\partial y} = \frac{-2y}{3z}$$

8. $V = \frac{1}{3}\pi R^2 \cdot h$

$$\frac{dV}{dt} = \frac{\partial V}{\partial R}\cdot\frac{dR}{dt} + \frac{\partial V}{\partial h}\cdot\frac{dh}{dt}$$

$$= \frac{2}{3}\pi h R \cdot \frac{dR}{dt} + \frac{1}{3}\pi R^2 \cdot \frac{dh}{dt}$$

$$= \frac{2}{3}\pi \cdot (140)(120)(1.8) + \frac{1}{3}\pi (120^2)(-2.5)$$

$$= 25,635 \text{ cm}^3/\text{s}$$

9. $I = V \cdot R^{-1}$

$$\frac{dI}{dt} = \frac{\partial I}{\partial V}\cdot\frac{dV}{dt} + \frac{\partial I}{\partial R}\cdot\frac{dR}{dt}$$

$$= R^{-1}\cdot\frac{dV}{dt} + -VR^{-2}\cdot\frac{dR}{dt}$$

$$= \left(\frac{1}{400}\right)(-0.01) - \frac{(400)(0.08)}{400^2} = -3.1\times10^{-5}\ \frac{A}{s}$$

10. $\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r}$

$= \frac{\partial z}{\partial x} \cos\theta + \frac{\partial z}{\partial y} \sin\theta$

$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial \theta}$

$= \frac{\partial z}{\partial x}(-r\sin\theta) + \frac{\partial z}{\partial y}(r\cos\theta)$

$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 =$

$\left(\frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta\right)^2 + \frac{1}{r^2}\left(-r\frac{\partial z}{\partial x}\sin\theta + r\frac{\partial z}{\partial y}\cos\theta\right)^2$

$= \left(\frac{\partial z}{\partial x}\right)^2\cos^2\theta + \left(\frac{\partial z}{\partial y}\right)^2\sin^2\theta + \left(\frac{\partial z}{\partial x}\right)^2\sin^2\theta + \left(\frac{\partial z}{\partial y}\right)\cos^2\theta$

$= \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$