PROBLEM SET 14

1. $D_u = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$

$= \left[3x^2y^4 + 4x^3y^3\right]\cos\theta + \left[4x^3y^3 + 3x^4y^2\right]\sin\theta$

$= 7\cdot\frac{\sqrt{3}}{2} + 7\cdot\frac{1}{2} = \frac{7}{2}\left(1+\sqrt{3}\right)$

2. a. $\nabla f = 2\cos(2x+3y)\,\hat{i} + 3\cos(2x+3y)\,\hat{j}$

b. $\nabla f = 2\hat{i} + 3\hat{j}$

c. $\nabla f \cdot u = 2\cdot\frac{\sqrt{3}}{2} + 3\cdot\frac{-1}{2} = \sqrt{3} - \frac{3}{2}$

3. a. $\nabla f = \frac{-y^2}{x^2}\,\hat{i} + \frac{2y}{x}\,\hat{j}$

b. $\nabla f = -4\hat{i} + 4\hat{j}$

c. $\nabla f \cdot u = -4(1) + 4\left(\frac{\sqrt{5}}{2}\right) = 2\sqrt{5} - 4$

4. $\vec{u} = \frac{1}{\sqrt{10}}\,\hat{i} + \frac{3}{\sqrt{10}}\,\hat{j}$

$D_u = f_p(p,q)\cdot u_p + f_q(p,q)\,u_q$

$= 4p^3 - 2pq^3 \cdot u_p + -3p^2q^2\cdot u_q$

$= \left[4\cdot 2^3 - 2\cdot 2\cdot 1^3\right]\cdot\frac{1}{\sqrt{10}} + \left[-3\cdot 2^2\cdot 1^2\right]\frac{3}{\sqrt{10}}$

$= \frac{28}{\sqrt{10}} - \frac{36}{\sqrt{10}} = \frac{-8}{\sqrt{10}}$

5. $\vec{u} = \frac{-3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$$D_u = f_x(x,y)\,u_x + f_y(x\,y)\,u_y$$
$$= e^x \cdot \sin(y) \cdot u_x + e^x \cdot \cos(y) \cdot u_y$$
$$= 1 \cdot \frac{\sqrt{3}}{2} \cdot \frac{-3}{5} + 1 \cdot \frac{1}{2} \cdot \frac{4}{5}$$
$$= \frac{2}{5} - \frac{3\sqrt{3}}{10}$$

6. $\nabla f = 2yx^{-\frac{1}{2}}\,\hat{i} + 4x^{\frac{1}{2}}\,\hat{j}$

$\nabla f(4,1) = \hat{i} + 8\hat{j}$ IS THE DIRECTION

$$\vec{u} = \frac{1}{\sqrt{65}}\hat{i} + \frac{8}{\sqrt{65}}\hat{j}$$

$\nabla f \cdot \vec{u} = \frac{1}{\sqrt{65}} + \frac{64}{\sqrt{65}} = \sqrt{65}$ IS THE MAXIMUM RATE OF CHANGE

7. a. $\vec{u} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} - \frac{\hat{k}}{\sqrt{3}}$

$$D_u = \nabla \cdot \vec{u} = [10x - 3y + yz]\frac{1}{\sqrt{3}} + [-3x + xz]\frac{1}{\sqrt{3}}$$
$$+ [xy] \cdot -\frac{1}{\sqrt{3}}$$
$$= \frac{32}{\sqrt{3}}$$

b. $\nabla V = 38\hat{i} + 6\hat{j} + 12\hat{k}$

c. $\vec{u} = \frac{38}{\sqrt{1624}}\hat{i} + \frac{6}{\sqrt{1624}}\hat{j} + \frac{12}{\sqrt{1624}}\hat{k}$

$$D_u = \nabla V \cdot \vec{u} = 2\sqrt{406}$$

8. $x^2 - y - z^2 = 0$

$\nabla F = 2x \cdot \hat{i} - \hat{j} - 2z\hat{k}$

$\nabla F(4,7,3) = 8\hat{i} - 7\hat{j} - 6\hat{k}$

$8(x-4) - 7(y-7) - 6(z-3) = 0$

$8x - 7y - 6z + 35 = 0$

THE NORMAL LINE HAS PARAMETRIC EQUATIONS:

$x = 3 + 8t$
$y = 7 - 7t$
$z = 3 - 6t$

9. $xyz^2 - 6 = 0$

$\nabla F = yz^2\hat{i} + xz^2\hat{j} + 2xyz\hat{k}$

$\nabla F(3,2,1) = 2\hat{i} + 3\hat{j} + 12\hat{k}$

$2(x-3) + 3(y-2) + 12(z-1) = 0$

$2x + 3y + 12z - 24 = 0$

NORMAL LINE:
$x = 3 + 2t$
$y = 2 + 3t$
$z = 1 + 12t$

10. $\nabla g = (2x-4)\,\hat{i} + 2y\,\hat{j}$

$\nabla g\,(1,2) = -2\,\hat{i} + 4\,\hat{j}$

$-2(x-1) + 4(y-2) = 0$

$x - 2y + 3 = 0$ IS THE TANGENT LINE EQN