PROBLEM SET 15

1. $f_x = 3x^2 - 12y = 0$ OR $x^2 = 4y$

$f_y = -12x + 24y^2 = 0$ OR $2y^2 = x$

$x = 2$ OR $x = 0$
$y = 1$ $y = 0$

$f(x,y) = -8$ $f(x,y) = 0$

$D = 432$ $D = -144$

$f_{xx} = 12$ $f_{xx} = 0$

MINIMUM SADDLE

2. $f_x = y - x^{-2} = 0$

$f_y = x - y^{-2} = 0$

$x = 1$
$y = 1$

$f(x,y) = 3$

$D = 3$

$f_{xx} = 1$

MINIMUM

3. BOUNDARY EXTREMA:

$y=1 \quad f(x,y) = 2x^2+5$ 5 IS MIN 7 IS MAX

$y=-1 \quad f(x,y) = 5$ 5 IS MIN 5 IS MAX

$x=1 \quad f(x,y) = y^2+y+5$ 4.75 IS MIN 7 IS MAX

$x=-1 \quad f(x,y) = y^2+y+5$ 4.75 IS MIN 7 IS MAX

INSIDE:

$f_x = 2x+2xy = 0$

$f_y = 2y+x^2 = 0$

$x=\sqrt{2}, \ y=-1$ OR $x=0, \ y=0$

$f(x,y) = 6$ $\quad f(x,y) = 4$

MINIMUM IS 4, MAXIMUM IS 7

4. $D = \sqrt{(x-2)^2+(y-0)^2+(z+3)^2}$

MINIMIZE D^2 WHERE

$D^2 = (x-2)^2+y^2+\left[(1-x-y)+3\right]^2$

$D^2 = (x-2)^2+y^2+(4-x-y)^2$

$f_x = 4x+2y-12 = 0$

$f_y = 4y+2x-8 = 0$

$y=\frac{2}{3} \quad x=\frac{8}{3} \quad z=-\frac{7}{3}$

$D = \frac{2}{\sqrt{3}}$

5. $V = l \cdot w \cdot h$

$c = 4l + 4h + 4w$

$\nabla V_l = wh \quad \nabla c_l = 4 \quad$ so $\quad wh = \lambda \cdot 4$

$\nabla V_w = lh \quad \nabla c_w = 4 \quad$ so $\quad lh = \lambda \cdot 4$

$\nabla V_h = l \cdot w \quad \nabla c_h = 4 \quad$ so $\quad lw = \lambda \cdot 4$

$l = w = h$ AND $\frac{c}{4} = l + h + w$

$l = \frac{c}{12} \quad w = \frac{c}{12} \quad h = \frac{c}{12}$

6. $\nabla f = -2x + 2y$
$\nabla c = \frac{1}{2}x + 2y$

$-2x = \lambda \cdot \frac{1}{2}x \qquad x = 0 \quad$ OR $\quad \lambda = -4$
$2y = \lambda \cdot 2y \qquad\qquad\qquad\qquad y = 0$

$\frac{1}{4}x^2 + y^2 = 1 \qquad y = \pm 1 \qquad x = \pm 2$

$f(0,-1) = 1$ minimum
$f(0,1) = 1$
$f(-2,0) = 4$ maximum
$f(2,0) = 4$

7. $\nabla f = y \cdot e^{xy} + x \cdot e^{xy}$

$\nabla c = 3x^2 + 3y^2$

$y \cdot e^{xy} = \lambda \cdot 3x^2$ $\qquad \dfrac{e^{xy}}{\lambda} = \dfrac{3x^2}{y} = \dfrac{3y^2}{x}$

$x \cdot e^{xy} = \lambda \cdot 3y^2$

$x^3 + y^3 = 16$ $\qquad x = y = 2$

$f(x,y) = e^4$ MAXIMUM

NO MINIMUM

8.

$x: \quad 1 = \lambda \cdot 1 + 0$ $\qquad x + y + z = 1$

$y: \quad 2 = \lambda \cdot 1 + \mu \cdot 2y$ $\qquad y^2 + z^2 = 4$

$z: \quad 0 = \lambda \cdot 1 + \mu \cdot 2z$

$\lambda = 1$ SO $\frac{1}{2} = \mu \cdot 2y$ AND $-\frac{1}{2} = \mu \cdot 2z$

$y = \dfrac{1}{2\mu}$ $\qquad z = -\dfrac{1}{2\mu}$

$$\left(\frac{1}{2\mu}\right)^2 + \left(\frac{1}{2\mu}\right)^2 = 4$$

$$\mu = \pm \frac{1}{2\sqrt{2}}$$

$y = \pm\sqrt{2}$ $\qquad z = \pm\sqrt{2}$ $\qquad x = 1$

$f(x,y,z) = 1 + 2\sqrt{2}$ MAXIMUM

$= 1 - 2\sqrt{2}$ MINIMUM

9. $2x + 4 = \lambda \cdot 2x$ $\qquad$ $2x(\lambda - 1) = 4$

$2y - 4 = \lambda \cdot 2y$ $\qquad$ $2y(\lambda - 1) = -4$

$$\frac{4}{2x} = \frac{-4}{2y}$$

$$x = -y$$

$x^2 + (-x)^2 \leq 9$

$x \leq \frac{3}{\sqrt{2}}$ so $x = \frac{3}{\sqrt{2}}$ $y = \frac{-3}{\sqrt{2}}$ OR $x = \frac{-3}{\sqrt{2}}$ $y = \frac{3}{\sqrt{2}}$

$f(x,y) = \frac{9}{2} + \frac{9}{2} + 4\left(\frac{3}{\sqrt{2}}\right) - 4\left(\frac{-3}{\sqrt{2}}\right) = 9 + \frac{24}{\sqrt{2}}$

MAXIMUM

FOR CRITICAL POINTS, USE

$f_x = 2x + 4 = 0$

$f_y = 2y - 4 = 0$

so $x = -2$ $y = 2$

$f(x,y) = -8$ MINIMUM

10. $P = xyz$

$100 = x + y + z$

$P_x = yz = \lambda \cdot 1$ $\qquad$ $x = y = z = \frac{100}{3}$

$P_y = xz = \lambda \cdot 1$

$P_z = xy = \lambda \cdot 1$