PROBLEM SET 17

1.

$$\int_0^1 \int_{x^2}^{x} (1+2y)\, dy \cdot dx =$$

$$\int_0^1 \Big[y+y^2 \Big]_{x^2}^{x} dx =$$

$$\int_0^1 x - x^4\, dx = \Big[\tfrac{1}{2}x^2 - \tfrac{1}{5}x^5 \Big]_0^1 = \frac{3}{10}$$

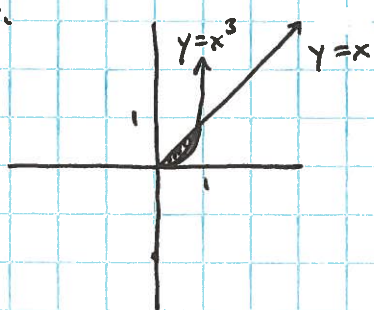
2.

$$\int_0^1 \int_0^{x^2} y\,(x^5+1)^{-1}\, dy \cdot dx =$$

$$\int_0^1 \Big[\frac{y^2}{2}(x^5+1)^{-1} \Big]_0^{x^2} = \int_0^1 \frac{x^4}{2}(x^5+1)^{-1}\, dx =$$

$$\frac{1}{10} \ln(x^5+1) \Big]_0^1 = \frac{\ln 2}{10}$$

3.



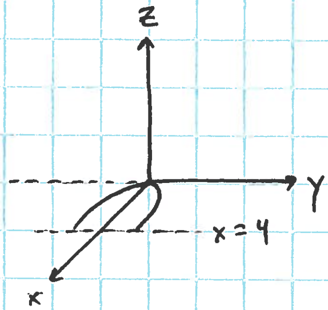
$$\int_0^1 \int_{x^3}^{x} (x^2+2y)\, dy \cdot dx =$$

$$\int_0^1 \Big[x^2 y + y^2 \Big]_{x^3}^{x} dx =$$

$$\int_0^1 x^3 + x^2 - x^5 - x^6\, dx =$$

$$\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 - \tfrac{1}{6}x^6 - \tfrac{1}{7}x^7 \Big]_0^1 = \frac{23}{84}$$

4.



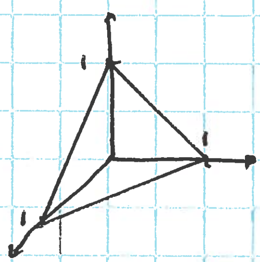
$$\int_{-2}^{2}\int_{0}^{y^2} 1 + x^2y^2 \, dx\, dy =$$

$$\int_{-2}^{2} \left[x + \frac{x^3y^2}{3} \right]_0^{y^2} dy =$$

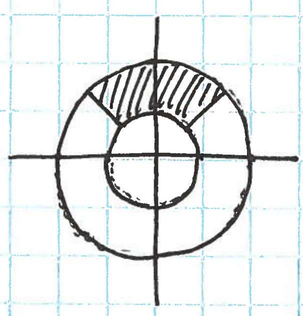
$$\int_{-2}^{2} y^2 + \frac{y^5}{3} \, dy =$$

$$\left[\frac{y^3}{3} + \frac{y^6}{18} \right]_{-2}^{2} = \frac{16}{3}$$

5.



6.

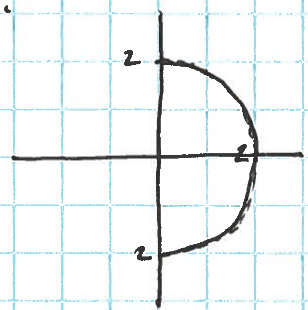


$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{1}^{2} R \cdot dR \cdot d\theta =$$

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left[\frac{1}{2} R^2 \right]_1^2 d\theta =$$

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{3}{2} \, d\theta = \frac{3}{2}\theta \Big]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{3\pi}{4}$$

7.



$$\iint e^{-x^2-y^2}\,dA =$$

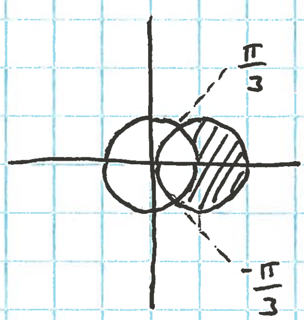
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 R\cdot e^{-R^2}\cdot dR\cdot d\theta =$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[-\tfrac{1}{2}e^{-R^2}\right]_0^2 d\theta =$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tfrac{1}{2}\left(1-e^{-4}\right) d\theta =$$

$$\tfrac{1}{2}\left(1-e^{-4}\right)\cdot\theta\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2}\left(1-e^{-4}\right)$$

8.



$(x-1)^2+y^2=1$

$x^2-2x+1+y^2=1$

$x^2+y^2=2x$

$R^2=2R\cos\theta$

$R=2\cos\theta$

$x^2+y^2=1$

$R=1$

$1=2\cos\theta$

$\theta=\pm\frac{\pi}{3}$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_1^{2\cos\theta} R\cdot dR\cdot d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[\tfrac{1}{2}R^2\right]_1^{2\cos\theta} d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2\cos^2\theta - \tfrac{1}{2}\cdot d\theta =$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(1+\cos 2\theta-\tfrac{1}{2}\right) d\theta = \tfrac{1}{2}\theta+\tfrac{1}{2}\sin(2\theta)\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \frac{\pi}{3}+\frac{\sqrt{3}}{2}$$

9. $z = 18 - 2(x^2 + y^2) = 18 - 2R^2$

$$V = \int_0^{2\pi} \int_0^3 (18 - R^2) R \cdot dR \cdot d\theta$$

$$= \int_0^{2\pi} \left[9R^2 - \frac{R^4}{4} \right]_0^3 d\theta = \int_0^{2\pi} \frac{243}{4} \, d\theta$$

$$= \frac{243}{4} \theta \Big]_0^{2\pi} = \frac{243\pi}{2}$$

10. $$\int_0^1 \int_y^{\sqrt{2-y^2}} (x + y) \, dx \cdot dy =$$

$$\int_0^1 \left[\frac{x^2}{2} + xy \right]_y^{\sqrt{2-y^2}} dy =$$

$$\int_0^1 1 - 2y^2 + y\sqrt{2 - y^2} \cdot dy =$$

$$\left[y - \frac{2}{3} y^3 - \frac{1}{3} (2 - y^2)^{\frac{3}{2}} \right]_0^1 = \frac{2\sqrt{2}}{3}$$