PROBLEM SET 19

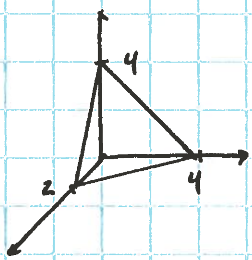
1.

$$\int_0^1 \int_x^{2x} \int_0^y (2xyz)\, dz \cdot dy \cdot dx =$$

$$\int_0^1 \int_x^{2x} (xy^3)\, dy \cdot dx =$$

$$\int_0^1 \frac{15}{4} x^5 \cdot dx = \left[\frac{15}{24} x^6\right]_0^1 = \frac{15}{24}$$

2.



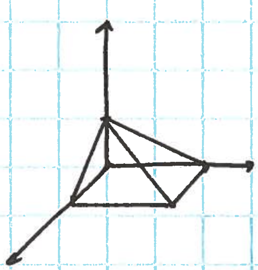
$$\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz \cdot dy \cdot dx =$$

$$\int_0^2 \int_0^{4-2x} (4-2x-y)\, dy \cdot dx =$$

$$\int_0^2 (8 - 8x + 2x^2)\, dx =$$

$$\left[8x - 4x^2 + \frac{2}{3} x^3\right]_0^2 = \frac{16}{3}$$

3.



4. TAKE, FOR EXAMPLE,

$$\iiint dx \cdot dy \cdot dz$$

THE BOUNDARIES FOR dx ARE IN TERMS OF y AND z:

$$x = \pm\sqrt{4-4z^2-y}$$

THE BOUNDARIES FOR dy ARE IN TERMS OF z WHERE $x=0$:

$$y = 0 \text{ AND } y = 4-4z^2$$

THE BOUNDARIES FOR dz ARE NUMERIC WHERE $x=0$ AND $y=0$:

$$z = \pm 1$$

a. $dx\,dy\,dz$:
$$-\sqrt{4-4z^2-y} \le x \le \sqrt{4-4z^2-y}$$
$$0 \le y \le 4-4z^2$$
$$-1 \le z \le 1$$

b. $dx\,dz\,dy$:
$$\sqrt{4-4z^2-y} \le x \le \sqrt{4-4z^2-y}$$
$$-\tfrac{1}{2}\sqrt{4-y} \le z \le \tfrac{1}{2}\sqrt{4-y}$$
$$0 \le y \le 4$$

c. $dy\,dx\,dz$:
$$0 \le y \le 4-x^2-4z^2$$
$$-2\sqrt{1-z^2} \le x \le 2\sqrt{1-z^2}$$
$$-1 \le z \le 1$$

d. $dy\,dz\,dx$:
$$0 \le y \le 4-x^2-4z^2$$
$$-\tfrac{1}{2}\sqrt{4-x^2} \le z \le \tfrac{1}{2}\sqrt{4-x^2}$$
$$-2 \le x \le 2$$

e. $dz\,dx\,dy$

$$-\tfrac{1}{2}\sqrt{4-x^2-y} \le z \le \tfrac{1}{2}\sqrt{4-x^2-y}$$
$$-\sqrt{4-y} \le x \le \sqrt{4-y}$$
$$0 \le y \le 4$$

f. $dz\,dy\,dx$

$$-\tfrac{1}{2}\sqrt{4-x^2-y} \le z \le \tfrac{1}{2}\sqrt{4-x^2-y}$$
$$0 \le y \le 4-x^2$$
$$-2 \le x \le 2$$

5a. $x = R\cos\theta = \sqrt{2}\cdot\cos\left(\frac{3\pi}{4}\right) = -1$

$y = R\sin\theta = \sqrt{2}\cdot\sin\left(\frac{3\pi}{4}\right) = 1$

$z = z = 2$

b. $R = \sqrt{x^2+y^2} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$z = z = -1$

6. $R^2 - R\cos\theta + z^2 = 1$

7. $$\int_0^{2\pi}\int_0^{2}\int_R^{4} z\cdot dz\cdot dR\cdot d\theta =$$

$$\int_0^{2\pi}\int_0^{2}\left(8-\tfrac{1}{2}R^2\right)dR\cdot d\theta =$$

$$\int_0^{2\pi}\left[8R-\tfrac{1}{6}R^3\right]_0^2 d\theta =$$

$$\int_0^{2\pi}\frac{44}{3}\,d\theta = \frac{88\pi}{3}$$

8a. $x = 2 \cdot \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4} = 1$

$y = 2 \cdot \sin\frac{\pi}{4} \cdot \sin\frac{\pi}{4} = 1$

$z = 2 \cdot \cos\frac{\pi}{4} = \sqrt{2}$

8b. $R = \sqrt{1^2 + 0^2 + 3} = 2$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$\phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

9. $R^2 - 2x = 0$

$R^2 - 2R\sin\phi\cos\theta = 0$

10. $$\int_0^{\pi}\int_0^{2\pi}\int_0^{5} R^2 \cdot R^2 \sin\phi \cdot dR \cdot d\theta \cdot d\phi =$$

$$\int_0^{\pi}\int_0^{2\pi} 625 \cdot \sin\phi \, d\theta \, d\phi$$

$$\int_0^{\pi} 625 \cdot 2\pi \cdot \sin\phi \, d\phi$$

$$= 1250\pi \left[-\cos\phi\right]_0^{\pi}$$

$$= 2500\pi$$