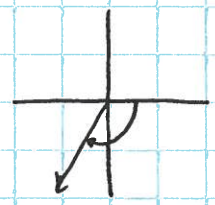
PROBLEM SET 2

1a.

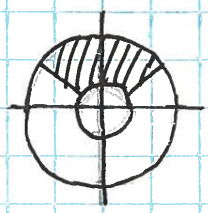


$x = -1$

$y = -\sqrt{3}$

1b. $\theta = \frac{2\pi}{3} \quad R = 2 \quad$ OR $\quad \theta = -\frac{\pi}{3} \quad R = -2$

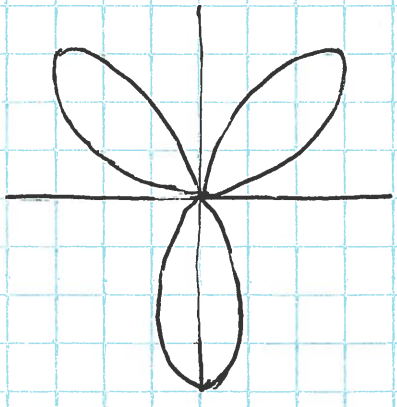
2.



3a. $x^2 + y^2 = 2x$

3b. $R\sin\theta = 1 + 3R\cos\theta$

4.



5. $R = 2 - \sin\left(\frac{\pi}{3}\right) = 2 - \frac{\sqrt{3}}{2}$ AND $\theta = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{\frac{dR}{d\theta}\sin\theta + R\cos\theta}{\frac{dR}{d\theta}\cos\theta - R\sin\theta} = \frac{-\cos\theta\sin\theta + R\cos\theta}{-\cos^2\theta - R\sin\theta}$$

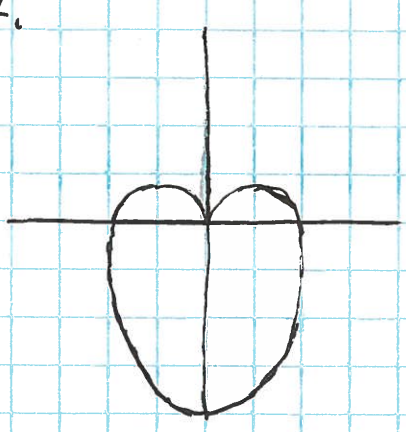
$$= \frac{1 - \frac{\sqrt{3}}{2}}{1 - \sqrt{3}}$$

6. $$A = \int_0^{\pi/6} \frac{1}{2} R^2 d\theta = \int_0^{\pi/6} \frac{1}{2} \cos^2\theta \cdot d\theta$$

$$= \frac{1}{4} \int_0^{\pi/6} (1 + \cos 2\theta) \cdot d\theta$$

$$= \frac{\pi}{24} + \frac{\sqrt{3}}{16}$$

7.



$$A = \int_0^{2\pi} \frac{1}{2} R^2 d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} (1 - \sin\theta)^2 d\theta$$

$$= \frac{3\pi}{2}$$

8. $3\cos(\theta) = 1 + \cos(\theta)$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$A = \int_{\frac{-\pi}{3}}^{\pi/3} \frac{1}{2}(3\cos\theta)^2 - \frac{1}{2}(1 + \cos\theta)^2 \cdot d\theta$$

$$A = \pi$$

9. $1+\sin\theta = 3\sin\theta$

$\sin\theta = \frac{1}{2}$ so $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$R = \frac{3}{2}$

10. $$L = \int_0^{\pi} \sqrt{R^2 + \left(\frac{dR}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{4\cos^2\theta + \left(-2\sin\theta\right)^2}\, d\theta$$

$= 2\pi$