

PROBLEM SET 5

$$1a. \quad a \cdot b = 12 - 10 - 3 = -1$$

$$a \cdot b = 2 - 1 - 0 = 1$$

$$2. \quad a \cdot b = 8 + 0 - 1 = 7$$

$$\cos \theta = \frac{7}{\sqrt{26} \cdot \sqrt{5}}$$

$$|a| = \sqrt{4^2 + 3^2 + 1^2} = \sqrt{26}$$

$$\theta = 52.125^\circ$$

$$|b| = \sqrt{5}$$

$$3. \quad \vec{PR} = \langle 1, 4 \rangle \quad \text{so } |PR| = \sqrt{17}$$

$$\vec{PQ} = \langle -2, 3 \rangle \quad \text{so } |PQ| = \sqrt{13}$$

$$PR \cdot PQ = -2 + 12 = 10$$

$$\cos \theta_p = \frac{10}{\sqrt{17} \cdot \sqrt{13}} \quad \theta_p = 47.726^\circ$$

$$\theta_r = 57.529^\circ$$

$$\theta_q = 74.745^\circ$$

$$4. \quad \text{LINES CROSS AT } (2, 1)$$

$$\langle 1, 2 \rangle \cdot \langle 1, -3 \rangle = -5 = \sqrt{5} \cdot \sqrt{10} \cdot \cos \theta$$

$$\theta = 135^\circ \quad \text{so } \theta_{\text{ACUTE}} = 55^\circ$$

$$5. \quad |R| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

DIRECTIONAL COSINES:

DIRECTIONAL ANGLES:

$$x: \frac{1}{\sqrt{14}}$$

$$y: \frac{-2}{\sqrt{14}}$$

$$z: \frac{-3}{\sqrt{14}}$$

$$\theta_x = 74.5^\circ$$

$$\theta_y = 122.3^\circ$$

$$\theta_z = 143.3^\circ$$

$$6. \quad \text{SCALAR COMPONENT} = \frac{A \cdot B}{|A|}$$

$$= \frac{3+12-6}{\sqrt{3^2+6^2+(-2)^2}} = \frac{9}{7}$$

$$\text{VECTOR PROJECTION} = \frac{A \cdot B}{|A|^2} \cdot \vec{A}$$

$$= \frac{9}{49} \cdot \langle 3, 6, -2 \rangle$$

$$= \left\langle \frac{27}{49}, \frac{54}{49}, \frac{-18}{49} \right\rangle$$

$$7. \quad \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ -1 & 0 & 5 \end{vmatrix} = 15i + 2j + 0k - [0i + 5j - 3k]$$

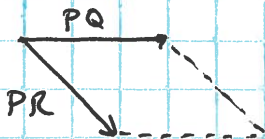
$$= 15i - 3j + 3k$$

$$A \cdot [A \times B] = 15 - 9 - 6 = 0$$

$$B \cdot [A \times B] = -15 + 0 + 15 = 0$$

$$8. \quad \begin{aligned} \vec{PQ} &= \langle -3, 1, 2 \rangle \\ \vec{PR} &= \langle 3, 2, 4 \rangle \end{aligned} \quad \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = 18j - 9k$$

$$\text{AREA OF } \triangle PQR = \frac{1}{2} \sqrt{18^2 + (-9)^2} = \frac{9\sqrt{5}}{2}$$



$$\text{So AREA}_{PQR} = \frac{9\sqrt{5}}{2}$$

$$9. \quad V = A \cdot [B \times C]$$

$$B \times C = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 2i + 8j - 3k$$

$$\langle 1, 2, 3 \rangle \cdot \langle 2, 8, -3 \rangle = 9$$

$$10. \quad \vec{PQ} = \langle 4, 2, 2 \rangle$$

$$\vec{PR} = \langle 3, 3, -1 \rangle$$

$$\vec{PS} = \langle 5, 5, 1 \rangle$$

$$\vec{PR} \times \vec{PS} = 8i - 8j$$

$$V = \langle 4, 2, 2 \rangle \cdot \langle 8, -8, 0 \rangle = 16$$