

Problem Set 1

1. For $x = t^2$ and $y = t^3 - 4t$ for $-3 \leq t \leq 3$, sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction of the curve as t increases.
2. For $x = t - 1$ and $y = t^3 + 1$ for $-2 \leq t \leq 2$, sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction of the curve as t increases. Then eliminate the parameter to find the Cartesian equation of the curve.
3. For $x = \frac{1}{2} \cos \theta$ and $y = 2 \sin \theta$ for $0 \leq \theta \leq \pi$, eliminate the parameter to find the Cartesian equation of the curve. Then sketch the curve and indicate with an arrow the direction of the curve as θ increases.
4. For $x = e^t - 1$ and $y = e^{2t}$, eliminate the parameter to find the Cartesian equation of the curve. Then sketch the curve and indicate with an arrow the direction of the curve as t increases.
5. For $x = \frac{1}{t}$ and $y = \sqrt{t} \cdot e^{-t}$, find $\frac{dy}{dx}$.
6. For the parametric equations $x = 1 + 4t - t^2$ and $y = 2 - t^3$, what is the equation for the tangent line to the curve at $t = 1$?
7. For the parametric equations $x = t^2 + 1$ and $y = e^t - 1$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For what values of t is the curve concave upward?
8. For the parametric equations $x = t^3 - 3t$ and $y = t^2 - 3$, find the points on the curve where:
 - (a) the tangent line is horizontal
 - (b) the tangent line is vertical
 - (c) the tangent line has a slope of 1
9. Find the area enclosed by the curve $x = t^2 - 2t$, $y = \sqrt{t}$ and the y-axis.
10. Find the length of the curve for $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.