Problem Set 12

1. Find the equation of the tangent plane to the surface of $z = 3y^2 - 2x^2 + x$ at the point (2, -1, -3).

2. Find the equation of the tangent plane to the surface of $z = x \cdot e^{xy}$ at the point (2, 0, 2).

3. Show that the function $f(x,y) = x^3y^4$ is differentiable at (1,1), then find the linearization L(x,y) of the function at that point.

4. Show that the function $f(x,y) = 1 + x \cdot \ln(xy - 5)$ is differentiable at (2,3), then find the linearization L(x,y) of the function at that point.

5. Find the differential of the function $z = e^{-2x}\cos(2\pi t)$.

6. Find the differential of the function $m = p^5q^3$.

7. If $z = 5x^2 + y^2$ and (x,y) changes from (1,2) to (1.05, 2.1), compare the values of Δz and dz.

8. The length and width of a rectangle are measured as 30cm and 24cm, respectively, with an error in measurement of, at most, 0.1cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

9. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4cm in diameter if the metal in the top and bottom is 0.1cm thick and the metal in the sides is 0.05cm thick.

10. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation PV = 8.31T, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in pressure if the volume increases from 12L to 12.3L and the temperature decreases from 310K to 305K.