Problem Set 14

1. For the function $f(x,y) = x^3y^4 + x^4y^3$, find the directional derivative of f at the point (1,1) in the direction of $\theta = \frac{\pi}{6}$.

2. For the function $f(x,y) = \sin(2x + 3y)$
(a) Find the gradient of f.
(b) Evaluate the gradient at point P(-6,4)
(c) Find the rate of change of f at P in the direction of $\mathbf{u} = \frac{1}{2}(\sqrt{3}\mathbf{i} - \mathbf{j})$

3. For the function $f(x,y) = \frac{y^2}{x}$
(a) Find the gradient of f.
(b) Evaluate the gradient at point P(1, 2)
(c) Find the rate of change of f at P in the direction of $\mathbf{u} = \frac{1}{2}(2\mathbf{i} + \sqrt{5}\mathbf{j})$

4. Find the directional derivative of the function $g(p,q) = p^4 - p^2q^3$ at the point (2,1) in the direction of $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$.

5. Find the directional derivative of the function $f(x,y) = e^x \cdot \sin(y)$ at the point $(0,\frac{\pi}{3})$ in the direction of $\mathbf{v} = \langle -6, 8 \rangle$

6. For the point (4,1) and the function $f(x,y) = 4y \cdot \sqrt{x}$, find the maximum rate of change of f and the direction in which it occurs.

7. Over a certain region of space, the electric potential V is given by
$V(x,y,z) = 5x^2 - 3xy + xyz$.
(a) Find the rate of change of the potential at P(3,4,5) in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
(b) In which direction does V change most rapidly at P?
(c) What is the maximum rate of change at P?

8. For the equation $y = x^2 - z^2$, find the equations of the tangent plane and then find the normal line to the given surface at the point (4, 7, 3).

9. For the equation $xyz^2 = 6$, find the equations of the tangent plane and then find the normal line to the given surface at the point (3, 2, 1).

10. If $g(x,y) = x^2 + y^2 - 4x$, find the gradient vector $\nabla g(1,2)$ and use it to find the tangent line to the level curve $g(x,y) = 1$ at the point (1,2). Sketch the level curve, the tangent line, and the gradient vector.