

## Problem Set 18

1. Electric charge is distributed over the rectangle  $0 \leq x \leq 5$ ,  $2 \leq y \leq 5$  so that the charge density is  $\sigma(x,y) = 2x + 4y$ . Find the total charge.
2. Find the mass and center of mass of the lamina that occupies  $D = \{(x,y) \mid 1 \leq x \leq 3, 1 \leq y \leq 4\}$  and has a density function  $\sigma(x,y) = ky^2$ .
3. Find the mass and center of mass of the triangular lamina with vertices  $(0,0)$ ,  $(2, 1)$ ,  $(0,3)$  and that has a density function  $\sigma(x,y) = x + y$ .
4. Find the mass and center of mass of the lamina bounded by  $y = 1 - x^2$  and  $y = 0$  and has a density function  $\sigma(x,y) = ky$ .
5. A lamina occupies the part of the disk  $x^2 + y^2 \leq 1$  in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis.
6. Find the moments of inertia  $I_x$ ,  $I_y$ , and  $I_o$  for the lamina bounded by  $y = 1 - x^2$  and  $y = 0$  and has a density function  $\sigma(x,y) = ky$ .
7. Find the moments of inertia  $I_x$  and  $I_y$  for a triangle with vertices  $(0,0)$ ,  $(b,0)$ , and  $(0,h)$  for a lamina with a constant density  $\sigma$ .
8. The joint density function for a pair of random variables  $X$  and  $Y$  is:  
 $f(x,y) = Cx(1 + y)$  if  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$  and  $f(x,y) = 0$  elsewhere.
  - (a) Find the value of the constant  $C$ .
  - (b) Find  $P(X \leq 1, Y \leq 1)$ .
  - (c) Find  $P(X + Y) \leq 1$ .
9. Find the surface area for the part of the plane  $z = 2 + 3x + 4y$  that lies above the rectangle  $[0,5] \times [1,4]$ .
10. Find the surface area for the part of the surface  $z = 1 + 3x + 2y^2$  that lies above the triangle with vertices  $(0,0)$ ,  $(0,1)$ , and  $(2,1)$ .